Optik - International Journal for Light and Electron Optics 240 (2021) 166634

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Optical fractional spherical magnetic flux flows with Heisenberg spherical Landau Lifshitz model

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A R T I C L E I N F O

Keywords:
$\mathbb{S}\alpha$-magnetic particle
Heisenberg spherical Landau Lifshitz model
Geometric magnetic flux density

PACS:
04.20.−q
03.50.De
02.40.−k

A B S T R A C T

In this paper, we first offer the approach of spherical magnetic Lorentz flux of spherical $\mathbb{S}\alpha$-magnetic particle flows by the spherical frame in spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$. Eventually, we obtain some optical conditions of spherical $\mathbb{S}\alpha$-magnetic Lorentz flux by using directional spherical fields. Moreover, we determine spherical magnetic Lorentz flux for spherical vector fields. Also, we give new construction for spherical curvatures of spherical $\mathbb{S}\alpha$-magnetic particle flows by considering Heisenberg spherical Landau Lifshitz model. Finally, magnetic flux surface is demonstrated in a static and uniform magnetic surface by using the analytical and numerical results with Heisenberg spherical Landau Lifshitz model.

1. Introduction

Differential geometric tools such as surfaces and curves have been appeared in many disciplines of theoretical and practical areas of science ranging from thermodynamics [1] to high energy strings [2], and from general relativity [3] to solitons [4], or even in plasma physics [5] and liquid crystals [6]. The motion of curves and the concept of the Frenet–Serret frame are the main common ingredient in all these applications.

These tools have also been considered in the research of magnetic structures significantly. Recently, many authors have focused on the subject of magnetic curves and investigate many important results. In these studies, one common approach has been used extensively. According to this approach, it is generally assumed that magnetic curves are trajectories of the time-independent moving charged particle on geometric manifolds or physical spacetime structures. This motion of the particle is specifically determined by the Lorentz force equation. Once the Lorentz force equation is managed to solve successfully, then many interesting characterizations have been developed from the geometric and physical points of view [7–15].

These structures implemented by many authors to define magnetic flux-tubes in the case of inflexional configuration and inflectional disequilibrium. In the presence of a magnetic field, the magnetic flux-tube is defined by the cylindrical thin tube of circular cross-section having a positive radius. The cases of twisted magnetic flux-tube and straight flux-tube are investigated separately in various studies. The geometric formulation of these tubes is derived by the Lorentz force equation and used to determine generic characterizations associated with the several useful applications to astrophysical flows, solar corona loops, etc. Nested toroidal flux surface is described due to the motion curves in magnetohydrostatic. It can be considered as a generalization of the magnetic flux-tube. All these results have been obtained through the Riemannian and non-Riemannian geometric data and facts [16–20].

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https://doi.org/10.1016/j.ijleo.2021.166634
Received 21 January 2021; Accepted 25 February 2021
Available online 24 March 2021

In recent times there have been diverse developments in the operation of geometric flux flows from computer perspective, which has both analytical and functional significance. Some researchers have desired to derive flux flows that take into account the optical surfaces of fields enclosed by flows magnetic particles [21–26].

The geometric phase investigation along the optical fiber investigation is mostly conducted by observing the action of electromagnetic particles and their features. Some nonlinear evolution structures are frequently encountered particularly in genuine-state physics, chemical physics, plasma physics, optical physics, fluid mechanics, etc. Even though these equations have been heavily used in many structures, it requires very hard work to obtain the explicit solutions of approximate systems. Thus, there exists no global or unified approach to demonstrate the exact solutions of all nonlinear transformation systems [27–37].

The aim of the present paper is to study the effect of the geometric interpretation of the notion of the Heisenberg spherical ferromagnetic spin for spherical flows of magnetic particles with the spherical-frame in spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$. Eventually, we obtain some optical conditions of spherical $\mathbb{S}\alpha-$ magnetic Lorentz flux by using spherical fields. Moreover, we determine spherical magnetic Lorentz flux for spherical fields. Also, we give new conditions for spherical potentials of spherical $\mathbb{S}\alpha-$ magnetic flows by considering Heisenberg spherical Landau Lifshitz model. Finally, the magnetic flux surface is demonstrated in a static and uniform magnetic surface by using the analytical and numerical results with Heisenberg spherical Landau Lifshitz model.

2. Backround on the spherical frame in $\mathbb{S}^2_{\mathbb{H}}$

Heisenberg metric g is given via

$$g = \mathrm{dx}^2 + \mathrm{dy}^2 + (\mathrm{dz} - \mathrm{xdy})^2.$$

Basis for Lie algebra of Heisenberg group is given by

$$\mathrm{e}_1 = \frac{\partial}{\partial x}, \quad \mathrm{e}_2 = \frac{\partial}{\partial y} + x\frac{\partial}{\partial z}, \quad \mathrm{e} = \frac{\partial}{\partial z}.$$

Let $\alpha : \mathbb{I} \rightarrow \mathbb{S}^2_{\mathbb{H}}$ be unit speed smooth particle. Spherical frame system is defined by spherical frame

$$\begin{bmatrix} \nabla_\sigma \alpha \\ \nabla_\sigma \mathrm{t} \\ \nabla_\sigma \mathrm{s} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & \varepsilon \\ 0 & -\varepsilon & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \mathrm{t} \\ \mathrm{s} \end{bmatrix},$$

where $\varepsilon = det(\alpha, \mathrm{t}, \nabla_\sigma \mathrm{t})$.

Also, the particle satisfying this spherical frame equations is defined Lorentzian spherical particle.

The vector products of spherical fields are given by

$$\alpha = \mathrm{t} \times \mathrm{s}, \quad \mathrm{t} = \mathrm{s} \times \alpha, \quad \mathrm{s} = \alpha \times \mathrm{t}.$$

3. Flows of $\mathbb{S}\boldsymbol{\alpha}$-magnetic particles in spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$

In this section, it is first described magnetic surfaces with the time evolution of the spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic particle in spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$. The time evolution is assumed to be a new design embedded in the spherical space. Thus, the fundamental geometric construction of the flows as surfaces can naturally be induced by moving spherical orthonormal fields.

♠ *Let α be smooth particle with magnetic field $\mathcal{B}$ in the spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$. We call particle α as a spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic particle if the spherical tangent field with the following equation:*

$$\nabla_\sigma \alpha = \phi(\alpha) = \mathcal{B} \times \alpha.$$

♠ *Lorentz force ϕ for spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic particle is given*

$$\begin{aligned} \phi(\boldsymbol{\alpha}) &= \pi_2 \varpi \mathrm{e}_1 - \pi_1 \varpi \mathrm{e}_2 + cos\varphi \mathrm{e}_3, \\ \phi(\mathrm{t}) &= -(\pi_1 + sin^3\varphi)\mathrm{e}_1 + (-\pi_2 + \omega\pi_4)\mathrm{e}_2 - (\pi_3 + sin^3\varphi)\mathrm{e}_3, \\ \phi(\mathrm{s}) &= -\omega\pi_2\varpi\mathrm{e}_1 + \omega\pi_1\varpi\mathrm{e}_2 - \omega cos\varphi \mathrm{e}_3, \end{aligned}$$

where $\omega = g(\phi(\mathrm{t}), \mathrm{s})$ *is a sufficiently smooth function. Also, magnetic field $\mathcal{B}$ is given by*

$$\mathcal{B} = (\pi_1\omega - \frac{1}{\varpi}sin^3\varphi)\mathrm{e}_1 + (\pi_2\omega + \chi_4)\mathrm{e}_2 + (\pi_3\omega - \frac{1}{\varpi}sin^3\varphi)\mathrm{e}_3,$$

where

$$\begin{aligned}
\pi_1 &= -\frac{1}{\varpi}sin\varphi cos[\varpi\sigma+\varpi_1],\\
\pi_2 &= \frac{1}{\varpi}sin\varphi sin[\varpi\sigma+\varpi_1],\\
\pi_3 &= (cos\varphi-\frac{1}{\varpi}sin^2\varphi)\sigma-\frac{1}{4\varpi^2}sin^2\varphi sin2[\varpi\sigma+\varpi_1],\\
\pi_4 &= (\frac{1}{\varpi}sin^2\varphi cos[\varpi\sigma+\varpi_1]cos\varphi+sin\varphi sin[\varpi\sigma+\varpi_1]\chi_3),\\
\varpi &= (\frac{\sqrt{1+\varepsilon^2}}{sin\varphi}-cos\varphi).
\end{aligned}$$

Let $\boldsymbol{\alpha}(\sigma,t)$ be the motion of regular spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic particle in spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$. The flow of spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic particle is given by

$$\nabla_t\boldsymbol{\alpha}=\chi_1\mathrm{t}+\chi_2\mathrm{s},$$

where χ_1,χ_2 are potentials of particle.

♠*Time derivatives of spherical frame*

$$\begin{aligned}
\nabla_t\boldsymbol{\alpha} &= (\chi_1\pi_2\varpi-\frac{\chi_2}{\varpi}sin^3\varphi)\mathrm{e}_1-(\chi_1\pi_1\varpi-\chi_2\pi_4)\mathrm{e}_2\\
&+(\chi_1cos\varphi-\frac{\chi_2}{\varpi}sin^3\varphi)\mathrm{e}_3,\\
\nabla_t\mathrm{t} &= -(\chi_1\pi_1+\frac{1}{\varpi}(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})sin^3\varphi)\mathrm{e}_1+(\pi_4(\chi_1\varepsilon\\
&+\frac{\partial\chi_2}{\partial\sigma})-\chi_1\pi_2)\mathrm{e}_2-(\chi_1\pi_3+\frac{1}{\varpi}(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})sin^3\varphi)\mathrm{e}_3,\\
\nabla_t\mathrm{s} &= -(\chi_2\pi_1+\pi_2\varpi(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma}))\mathrm{e}_1+(\pi_1\varpi(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})\\
&-\chi_2\pi_2)\mathrm{e}_2-(\chi_2\pi_3+cos\varphi(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma}))\mathrm{e}_3.
\end{aligned}$$

♠*Flows of $\phi(\boldsymbol{\alpha}),\phi(\mathrm{t}),\phi(\mathrm{s})$ forces of the spherical frame are presented by*

$$\begin{aligned}
\nabla_t\phi(\boldsymbol{\alpha}) &= -(\chi_1\pi_1+\frac{1}{\varpi}(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})sin^3\varphi)\mathrm{e}_1-(\chi_1\pi_2-\pi_4(\chi_1\varepsilon\\
&+\frac{\partial\chi_2}{\partial\sigma}))\mathrm{e}_2-(\chi_1\pi_3+\frac{1}{\varpi}(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})sin^3\varphi)\mathrm{e}_3,\\
\nabla_t\phi(\mathrm{t}) &= -((\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})+\chi_1)\pi_2\varpi+\omega\chi_2\pi_1+\frac{1}{\varpi}(\frac{\partial\omega}{\partial t}-\chi_2)sin^3\varphi)\mathrm{e}_1\\
&+((\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})+\chi_1)\pi_1\varpi-\omega\chi_2\pi_2+\pi_4(\frac{\partial\omega}{\partial t}-\chi_2))\mathrm{e}_2\\
&-((\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})+\chi_1)cos\varphi+\omega\chi_2\pi_3+\frac{1}{\varpi}(\frac{\partial\omega}{\partial t}-\chi_2)sin^3\varphi)\mathrm{e}_3,\\
\nabla_t\phi(\mathrm{s}) &= (\frac{\omega}{\varpi}(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})sin^3\varphi-\pi_2\varpi\frac{\partial\omega}{\partial t}+\omega\chi_1\pi_1)\mathrm{e}_1+(\pi_1\varpi\frac{\partial\omega}{\partial t}+\omega\chi_1\pi_2\\
&-\pi_4\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma}))\mathrm{e}_2+(\frac{\omega}{\varpi}(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})sin^3\varphi+\omega\chi_1\pi_3-cos\varphi\frac{\partial\omega}{\partial t})\mathrm{e}_3.
\end{aligned}$$

4. Spherical magnetic Lorentz flux surfaces

The magnetic flux equation theory or the theory of flux systems has had an enormous impact on applied physical and mathematical studies. This framework combines some subtle techniques in nonlinear flux optics, field designs and sigma models, fluid dynamics, relativity, electromagnetic wave theory. In this section, we obtain spherical magnetic $\phi(\boldsymbol{\alpha}),\phi(\mathrm{t}),\phi(\mathrm{s})$ flux conditions by using the Heisenberg spherical Landau Lifshitz model.

Case 1. Spherical magnetic $\phi(\boldsymbol{\alpha})$ flux with Heisenberg spherical Landau Lifshitz model

Theorem 4.1. *The spherical Heisenberg magnetic $\phi(\boldsymbol{\alpha})$ flux${}^m\mathcal{F}^{\mathcal{LL}}_{\phi(\boldsymbol{\alpha})}$ is given by*

$$
{}^{m}\mathcal{F}^{\mathcal{LL}}_{\phi(\boldsymbol{\alpha})} = \int_{\mathcal{F}}(-\pi_1\varpi\varepsilon(\pi_2\omega+\chi_4)\frac{\partial\varepsilon}{\partial\sigma}+\pi_2\varpi\varepsilon(\pi_1\omega - \frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}+cos\varphi\varepsilon(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma})\mathrm{dv}.
$$

Proof. From definition of spherical Heisenberg magnetic $\phi(\boldsymbol{\alpha})$ flux

$$
{}^{m}\mathcal{F}_{\phi(\boldsymbol{\alpha})} = \int_{\mathcal{F}}\mathcal{B}\cdot\nabla_s\phi(\boldsymbol{\alpha})\times\nabla_t\phi(\boldsymbol{\alpha})\mathrm{dv}.
$$

By short calculations, we have

$$
\nabla_\sigma\phi(\boldsymbol{\alpha})\times\nabla_t\phi(\boldsymbol{\alpha}) = \pi_2\varpi\frac{\partial\chi_2}{\partial\sigma}\mathrm{e}_1-\pi_1\varpi\frac{\partial\chi_2}{\partial\sigma}\mathrm{e}_2+cos\varphi\frac{\partial\chi_2}{\partial\sigma}\mathrm{e}_3.
$$

Magnetic flux density of $\phi(\mathrm{t_q})$ is given by

$$
{}^{m}\mathcal{L}_{\phi(\boldsymbol{\alpha})} = \pi_2\varpi(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\chi_2}{\partial\sigma}-\pi_1\varpi(\pi_2\omega+\chi_4)\frac{\partial\chi_2}{\partial\sigma}+cos\varphi(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi).
$$

Moreover, $\phi(\boldsymbol{\alpha})$ flux is obtained in the following way

$$
{}^{m}\mathcal{F}_{\phi(\boldsymbol{\alpha})} = \int_{\mathcal{F}}(-\pi_1\varpi(\pi_2\omega+\chi_4)\frac{\partial\chi_2}{\partial\sigma}+\pi_2\varpi(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\chi_2}{\partial\sigma}+cos\varphi(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi))\mathrm{dv}.
$$

From Landau Lifshitz model, flux density is given by

$$
{}^{m}\mathcal{L}^{\mathcal{LL}}_{\phi(\boldsymbol{\alpha})} = \mathcal{B}\cdot\nabla_s\phi(\boldsymbol{\alpha})\times\phi(\boldsymbol{\alpha})\times\nabla_s^2\phi(\boldsymbol{\alpha}).
$$

Also, we get

$$
\phi(\boldsymbol{\alpha})\times\nabla_\sigma^2\phi(\boldsymbol{\alpha}) = \pi_1\frac{\partial\varepsilon}{\partial\sigma}\mathrm{e}_1+\pi_2\frac{\partial\varepsilon}{\partial\sigma}\mathrm{e}_2+\pi_3\frac{\partial\varepsilon}{\partial\sigma}\mathrm{e}_3.
$$

Similarly, we can obtain that

$$
\nabla_\sigma\phi(\boldsymbol{\alpha})\times\phi(\boldsymbol{\alpha})\times\nabla_\sigma^2\phi(\boldsymbol{\alpha}) = \pi_2\varpi\varepsilon\frac{\partial\varepsilon}{\partial\sigma}\mathrm{e}_1-\pi_1\varpi\varepsilon\frac{\partial\varepsilon}{\partial\sigma}\mathrm{e}_2+cos\varphi\varepsilon\frac{\partial\varepsilon}{\partial\sigma}\mathrm{e}_3.
$$

Spherical Landau Lifshitz magnetic $\phi(\boldsymbol{\alpha})$ flux is given by

$$
{}^{m}\mathcal{L}^{\mathcal{LL}}_{\phi(\boldsymbol{\alpha})} = \pi_2\varpi\varepsilon(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}-\pi_1\varpi\varepsilon(\pi_2\omega+\chi_4)\frac{\partial\varepsilon}{\partial\sigma}+cos\varphi\varepsilon(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}.
$$

From above equation, we have

$$
{}^{m}\mathcal{F}^{\mathcal{LL}}_{\phi(\boldsymbol{\alpha})} = \int_{\mathcal{F}}(-\pi_1\varpi\varepsilon(\pi_2\omega+\chi_4)\frac{\partial\varepsilon}{\partial\sigma}+\pi_2\varpi\varepsilon(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}+cos\varphi\varepsilon(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma})\mathrm{dv}.
$$

This proves the theorem.

In the light of Theorem 3.1, we express the following important results.

- *The Heisenberg magnetic $\phi(\boldsymbol{\alpha})$ flux surface condition is given by*

$$
\pi_2\varpi(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\chi_2}{\partial\sigma}-\pi_1\varpi(\pi_2\omega+\chi_4)\frac{\partial\chi_2}{\partial\sigma}+cos\varphi(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)=0.
$$

- *The Heisenberg magnetic $\phi(\boldsymbol{\alpha})$ Landau Lifshitz flux surface is presented by*

$$
\pi_2\varpi\varepsilon(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}-\pi_1\varpi\varepsilon(\pi_2\omega+\chi_4)\frac{\partial\varepsilon}{\partial\sigma}+cos\varphi\varepsilon(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}=0.
$$

Therefore, we have the following corollary.

Corollary 1. *Gauss's law of the $\phi(\boldsymbol{\alpha})$ Landau Lifshitz flux closed surface is presented by*

$$
\oint_S(-\pi_1\varpi\varepsilon(\pi_2\omega+\chi_4)\frac{\partial\varepsilon}{\partial\sigma}+\pi_2\varpi\varepsilon(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma}+cos\varphi\varepsilon(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)\frac{\partial\varepsilon}{\partial\sigma})\mathrm{dv}=0.
$$

For numerical work, the above systems are often used spherical magnetic flux density. Spherical magnetic $\phi(\boldsymbol{\alpha})$ Landau Lifshitz flux is usually derived using the properties of the divergence-free nature of the magnetic field along with the definition of the spherical magnetic flux density in spherical Heisenberg space $\mathbb{S}^2_{\mathbb{H}}$. To obtain the visualization of the evolved systems of the magnetic $\phi(\boldsymbol{\alpha})$ flux ${}^{m}\mathcal{F}_{\phi(\boldsymbol{\alpha})}$ we use the basic numerical algorithms to solve the above equations at Matlab and Comsol software. This approach presents the

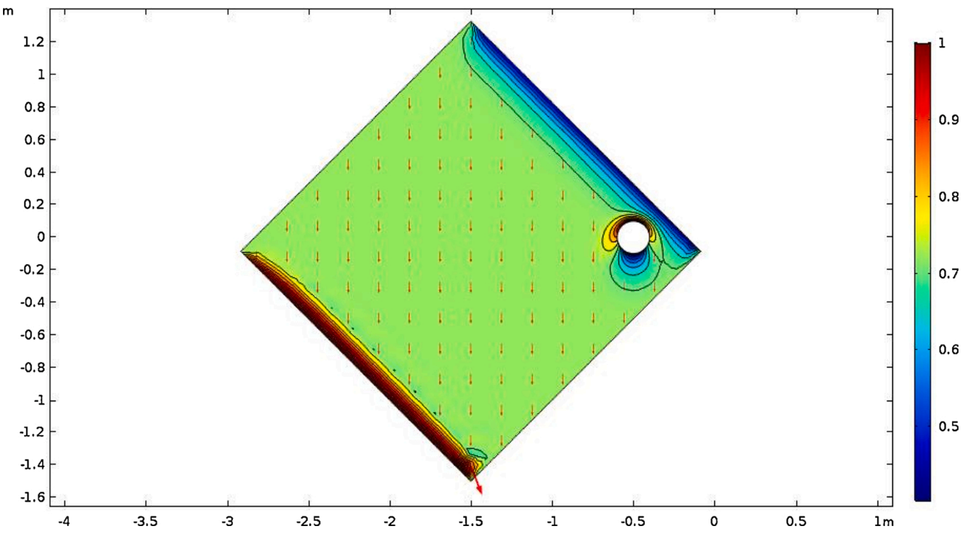

Fig. 1. Spherical magnetic $\phi(\alpha)$ Landau Lifshitz flux with ideal conductor.

results of the magnetic $\phi(\boldsymbol{\alpha})$ flux ${}^{m}\mathcal{F}_{\phi(\boldsymbol{\alpha})}$ on the spherical magnetic flux density and magnetic field of an ideal conductor at minimum density are illustrated in Fig. 1.

Case 2. Spherical Magnetic $\phi(\mathrm{t})$ flux with Heisenberg spherical Landau Lifshitz model

Theorem 4.2. *The spherical Heisenberg magnetic $\phi(\mathrm{t})$ Landau Lifshitz flux ${}^{m}\mathcal{F}^{\mathcal{LL}}_{\phi(\mathrm{t})}$ is given by*

$$
\begin{aligned}
{}^{m}\mathcal{F}^{\mathcal{LL}}_{\phi(\mathrm{t})} = & \int_{\mathcal{F}}((\pi_2\omega+\chi_4)(\pi_4\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\pi_1\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon \\
& +2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_2)+(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_1 \\
& -\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)(cos\varphi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon \\
& +2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi-\eta_2\pi_3))\mathrm{dv}.
\end{aligned}
$$

Proof. From Lorentz force equation, we have

$$
\begin{aligned}
\nabla_\sigma\phi(\mathrm{t})\times\nabla_t\phi(\mathrm{t}) = & (\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_1-\pi_2\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma})\mathrm{e}_1 \\
& +(\pi_1\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma}-\pi_4(\omega\varepsilon+1)\omega\chi_2+\eta_1\pi_2)\mathrm{e}_2 \\
& +(\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_3-cos\varphi\omega\chi_2\frac{\partial\omega}{\partial\sigma})\mathrm{e}_3,
\end{aligned}
$$

where

$$
\eta_1=\frac{\partial\omega}{\partial\sigma}(\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})+\chi_1)-(\omega\varepsilon+1)(\frac{\partial\omega}{\partial t}-\chi_2).
$$

$$
\begin{aligned}
{}^{m}\mathcal{L}_{\phi(\mathrm{t})} = & (\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_1-\pi_2\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma})(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi) \\
& +(\pi_2\omega+\chi_4)(\pi_1\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma}-\pi_4(\omega\varepsilon+1)\omega\chi_2+\eta_1\pi_2) \\
& +(\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_3-cos\varphi\omega\chi_2\frac{\partial\omega}{\partial\sigma})(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi).
\end{aligned}
$$

Then, ${}^{m}\mathcal{F}_{\phi(\mathrm{t})}$ is given by

$$\begin{aligned} {}^{m}\mathcal{F}_{\phi(\mathrm{t})} \;=\; & \int_{\mathcal{F}}((\pi_1\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma}-\pi_4(\omega\varepsilon+1)\omega\chi_2+\eta_1\pi_2)(\pi_2\omega+\chi_4) \\ & +(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_1-\pi_2\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma}) \\ & +(\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_3-cos\varphi\omega\chi_2\frac{\partial\omega}{\partial\sigma})(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi))\mathrm{dv}. \end{aligned}$$

In a similar way, we get

$$\begin{aligned} \nabla_\sigma\phi(\mathrm{t})\times\phi(\mathrm{t})\times\nabla_\sigma^2\phi(\mathrm{t}) = & (\pi_2\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_1-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma} \\ & +2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)\mathrm{e}_1+(\pi_4\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\pi_1\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon) \\ & -\eta_2\pi_2)\mathrm{e}_2+(cos\varphi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi-\eta_2\pi_3)\mathrm{e}_3, \end{aligned}$$

where

$$\eta_2=((\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)(\omega\varepsilon+1)-\frac{\partial\omega}{\partial\sigma}(\frac{\partial^2\omega}{\partial\sigma^2}+(\omega\varepsilon+1)(\omega-\varepsilon))).$$

From above equations, we get

$$\begin{aligned} {}^{m}\mathcal{L}^{\mathcal{LL}}_{\phi(\mathrm{t})} = & (\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_1-\frac{1}{\varpi}\omega(\omega\varepsilon \\ & +1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_2\omega+\chi_4)(\pi_4\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon) \\ & -\pi_1\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_2)+(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)(cos\varphi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon \\ & +2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi-\eta_2\pi_3). \end{aligned}$$

Similarly, magnetic $\phi(\mathrm{t})$ Landau Lifshitz flux is given by

$$\begin{aligned} {}^{m}\mathcal{F}^{\mathcal{LL}}_{\phi(\mathrm{t})} = & \int_{\mathcal{F}}((\pi_2\omega+\chi_4)(\pi_4\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\pi_1\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon \\ & +2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_2)+(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_1 \\ & -\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)(cos\varphi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon \\ & +2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi-\eta_2\pi_3))\mathrm{dv}, \end{aligned}$$

which completes the proof of the theorem.

In the light of Theorem 3.2, we express the following important results.

- *The Heisenberg magnetic $\phi(\mathrm{t})$ flux surface condition is given by*

$$\begin{aligned} & (\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_1-\pi_2\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma})(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi) \\ & +(\pi_2\omega+\chi_4)(\pi_1\varpi\omega\chi_2\frac{\partial\omega}{\partial\sigma}-\pi_4(\omega\varepsilon+1)\omega\chi_2+\eta_1\pi_2) \\ & +(\frac{1}{\varpi}(\omega\varepsilon+1)\omega\chi_2 sin^3\varphi+\eta_1\pi_3-cos\varphi\omega\chi_2\frac{\partial\omega}{\partial\sigma})(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)=0. \end{aligned}$$

- *The Heisenberg magnetic $\phi(\mathrm{t})$ Landau Lifshitz flux surface condition is given by*

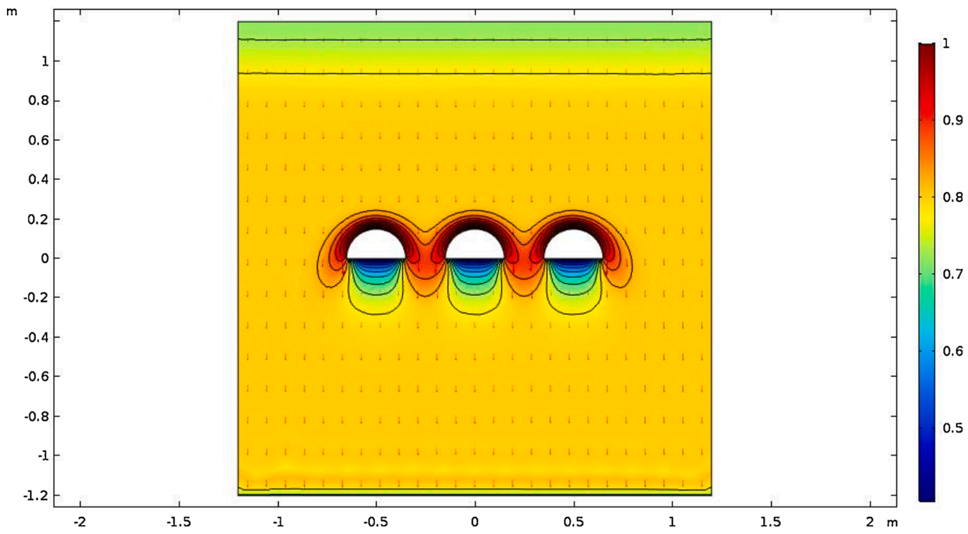

Fig. 2. Spherical magnetic ϕ(t) Landau Lifshitz flux with ideal conductor.

$$(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_1-\frac{1}{\varpi}\omega(\omega\varepsilon$$
$$+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_2\omega+\chi_4)(\pi_4\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)$$
$$-\pi_1\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_2)+(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)(cos\varphi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi-\eta_2\pi_3)=0.$$

Therefore, we have the following corollary.

Corollary 2. *Gauss's law of the ϕ(t) Landau Lifshitz flux closed surface is presented by*

$$\oint_S((\pi_2\omega+\chi_4)(\pi_4\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\pi_1\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_2)+(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\eta_2\pi_1$$
$$-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)(cos\varphi\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial}{\partial\sigma}\varepsilon$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{1}{\varpi}\omega(\omega\varepsilon+1)(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi-\eta_2\pi_3))\mathrm{dv}.$$

For numerical work, the above systems are often used spherical magnetic flux density. Spherical magnetic ϕ(t) Landau Lifshitz flux is usually derived using the properties of the divergence-free nature of the magnetic field along with the definition of the spherical magnetic flux density. To obtain the visualization of the evolved systems of the magnetic ϕ(t) flux ${}^m\mathcal{F}_{\phi(t)}$ we use the basic numerical algorithms to solve the above equations at Matlab and Comsol software. This approach presents the results of the magnetic ϕ(t) flux ${}^m\mathcal{F}_{\phi(t)}$ on the spherical magnetic flux density and magnetic field of an ideal conductor at minimum density are illustrated in Fig. 2.

Case 3. Spherical magnetic ϕ(s) flux with Heisenberg spherical Landau Lifshitz model

Theorem 4.3. *The spherical Heisenberg magnetic ϕ(s) Landau Lifshitz flux ${}^m\mathcal{F}^{\mathcal{LL}}_{\phi(s)}$ is given by*

$${}^m\mathcal{F}^{\mathcal{LL}}_{\phi(s)}=\int_{\mathcal{F}}((\pi_2\omega+\chi_4)(\pi_1\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})-2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_2$$
$$+\pi_4\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon))-(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})$$
$$+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_1+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)-(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)((\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})cos\varphi+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_3))\mathrm{dv}.$$

Proof. By some calculations, we have

$$\nabla_\sigma\phi(\mathrm{s})\times\nabla_t\phi(\mathrm{s})=(\pi_1(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})+\pi_2\varpi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)$$
$$-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)\mathrm{e}_1+(\pi_2(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})-\pi_1\varpi(\omega^2(\chi_1\varepsilon$$
$$+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)+\pi_4(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t}))(\pi_2\omega+\chi_4)+(\pi_3(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})$$
$$-\omega\varepsilon\frac{\partial\omega}{\partial t})+cos\varphi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)\mathrm{e}_3.$$

Flux density of $\phi(\mathrm{s})$ is given by

$${}^m\mathcal{L}_{\phi(\mathrm{s})}=(\pi_1(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})+\pi_2\varpi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1$$
$$-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)+(\pi_2(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})-\pi_1\varpi(\omega^2(\chi_1\varepsilon$$
$$+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)+\pi_4(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t}))\mathrm{e}_2+(\pi_3(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})$$
$$+cos\varphi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi).$$

Using above equation in phase, we obtain that

$${}^m\mathcal{F}_{\phi(\mathrm{s})}=\int_{\mathcal{F}}((\pi_2(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})-\pi_1\varpi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)$$
$$+\pi_4(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t}))(\pi_2\omega+\chi_4)+(\pi_1(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})+\pi_2\varpi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})$$
$$-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)+(\pi_3(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})$$
$$+cos\varphi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi))\mathrm{dv}.$$

Also, Landau Lifshitz model for $\phi(\mathrm{s})$, we get that

$${}^m\mathcal{L}^{\mathcal{LL}}_{\phi(\mathrm{s})}=\mathcal{B}\cdot\nabla_s\phi(\mathrm{s})\times\phi(\mathrm{s})\times\nabla^2_s\phi(\mathrm{s}).$$

By using the relations of Lorentz force, we get

$$\phi(\mathrm{s})\times\nabla^2_\sigma\phi(\mathrm{s})=(\pi_1\omega(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{2\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}sin^3\varphi)\mathrm{e}_1+(\pi_2\omega(\omega\frac{\partial\varepsilon}{\partial\sigma}$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega\pi_4\frac{\partial\omega}{\partial\sigma})\mathrm{e}_2+(\pi_3\omega(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)-\frac{2\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}sin^3\varphi)\mathrm{e}_3.$$

On the other hand, cross product are

$$\nabla_\sigma\phi(\mathrm{s})\times\phi(\mathrm{s})\times\nabla^2_\sigma\phi(\mathrm{s})=-(\pi_2\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_1$$
$$+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)\mathrm{e}_1+(\pi_1\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})$$
$$-2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_2+\pi_4\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon))\mathrm{e}_2-((\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)$$
$$+2\omega^2\frac{\partial\omega}{\partial\sigma})cos\varphi+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_3)\mathrm{e}_3.$$

Also, we find that

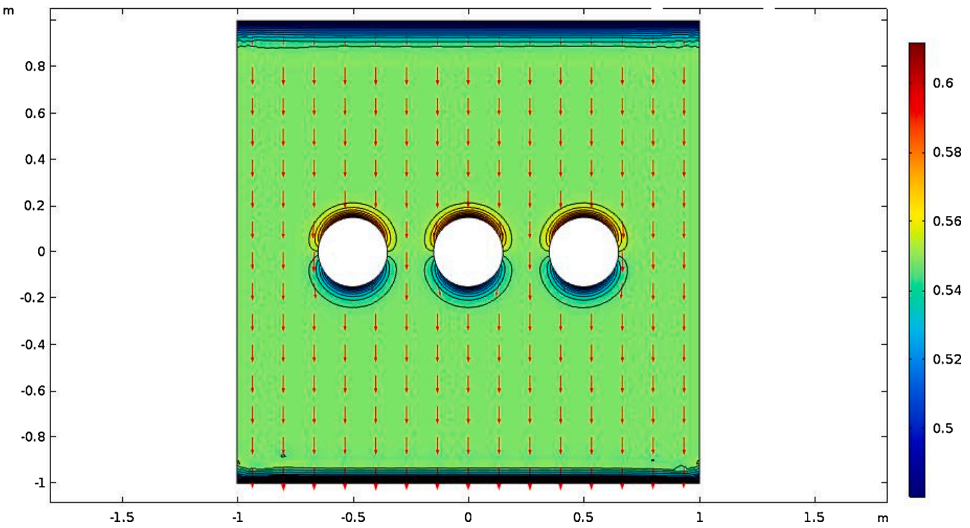

Fig. 3. Spherical magnetic $\phi(\mathrm{s})$ Landau Lifshitz flux with ideal conductor.

$$\begin{aligned}
{}^{m}\mathcal{L}^{\mathcal{LL}}_{\phi(\mathrm{s})} &= -(\pi_1\omega - \frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_1 \\
&+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_2\omega+\chi_4)(\pi_1\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma}) \\
&-2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_2+\pi_4\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon))-(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)((\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma} \\
&+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})cos\varphi+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_3).
\end{aligned}$$

Thus, we immediately obtain that

$$\begin{aligned}
{}^{m}\mathcal{F}^{\mathcal{LL}}_{\phi(\mathrm{s})} &= \int_{\mathcal{F}}((\pi_2\omega+\chi_4)(\pi_1\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})-2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_2 \\
&+\pi_4\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon))-(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma}) \\
&+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_1+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)-(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)((\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma} \\
&+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})cos\varphi+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_3))\mathrm{dv}.
\end{aligned}$$

This proves the theorem.

In the light of Theorem 3.3, we express the following important results.

- *The Heisenberg magnetic $\phi(\mathrm{s})$ flux surface condition is given by*

$$\begin{aligned}
&(\pi_1(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})+\pi_2\varpi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1 \\
&-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)+(\pi_2(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t})-\pi_1\varpi(\omega^2(\chi_1\varepsilon \\
&+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)+\pi_4(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t}))\mathrm{e}_2+(\pi_3(\frac{\partial\omega}{\partial\sigma}\omega(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\omega\varepsilon\frac{\partial\omega}{\partial t}) \\
&+cos\varphi(\omega^2(\chi_1\varepsilon+\frac{\partial\chi_2}{\partial\sigma})-\varepsilon\omega^2\chi_1)-\frac{1}{\varpi}(\frac{\partial\omega}{\partial\sigma}\omega\chi_1-\omega\frac{\partial\omega}{\partial t})sin^3\varphi)(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)=0.
\end{aligned}$$

- *The Heisenberg magnetic $\phi(\mathrm{s})$ Landau Lifshitz flux surface condition is given by*

$$-(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_1$$
$$+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)+(\pi_2\omega+\chi_4)(\pi_1\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})$$
$$-2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_2+\pi_4\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon))-(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)((\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})cos\varphi+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_3)=0.$$

Finally, we have the following corollary.

Corollary 3. *Gauss's law of the $\phi(\mathrm{s})$ Landau Lifshitz flux closed surface is presented by*

$$\oint_S((\pi_2\omega+\chi_4)(\pi_1\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})-2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_2$$
$$+\pi_4\omega\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon))-(\pi_1\omega-\frac{1}{\varpi}sin^3\varphi)(\pi_2\varpi(\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})$$
$$+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_1+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi)-(\pi_3\omega-\frac{1}{\varpi}sin^3\varphi)((\varepsilon\omega^2(\omega\frac{\partial\varepsilon}{\partial\sigma}$$
$$+2\frac{\partial\omega}{\partial\sigma}\varepsilon)+2\omega^2\frac{\partial\omega}{\partial\sigma})cos\varphi+\frac{\omega}{\varpi}\frac{\partial\omega}{\partial\sigma}(\omega\frac{\partial\varepsilon}{\partial\sigma}+2\frac{\partial\omega}{\partial\sigma}\varepsilon)sin^3\varphi+2\omega(\frac{\partial\omega}{\partial\sigma})^2\pi_3))\mathrm{dv}=0.$$

For numerical work, the above systems are often used spherical magnetic flux density. Spherical magnetic $\phi(\mathrm{s})$ Landau Lifshitz flux is usually derived using the properties of the divergence-free nature of the magnetic field along with the definition of the spherical magnetic flux density. To obtain the visualization of the evolved systems of the magnetic $\phi(\mathrm{s})$ flux ${}^m\mathcal{F}_{\phi(\mathrm{s})}$ we use the basic numerical algorithms to solve the above equations at Matlab and Comsol software. This approach presents the results of the magnetic $\phi(\mathrm{s})$ flux ${}^m\mathcal{F}_{\phi(\mathrm{s})}$ on the spherical magnetic flux density and magnetic field of an ideal conductor at minimum density are illustrated in Fig. 3.

5. Application to fractional calculus

In this section, the connection between the Laplacian-like non-linear equation the celebrated Heisenberg Landau Lifshitz model flow is investigated in the spherical magnetic flux density and spherical flow lines. For magnetic $\phi(\boldsymbol{\alpha})$ flux surface, we have already induced solitonic equations that are associated with curves of geometric quantities. If one considers the appropriate limiting and scaling process then a basic geometric derivation admits the following reciprocal transformation

$$\frac{\partial^\theta\varepsilon(\sigma,t)}{\partial t^\theta}-\omega\frac{\partial^2\varepsilon(\sigma,t)}{\partial\sigma^2}+\varepsilon(\sigma,t)^2\chi=0, \tag{1}$$

where $\frac{\partial^\theta}{\partial t^\theta}$ is the conformable derivative operator; ω and χ are real valued constants. In [38]; scientists studied conformable type of fractional derivative in 2014 as a new definition of local fractional operator. It's very easier to work with this fractional derivative. Recently, several studies have been done relatedto conformable type of fractional calculations [39,40].

The definition of conformable fractional derivative of order $\theta\in(0,1)$ defined as the following expression [38],

$${}_tD^\theta f(t)=\lim_{\varepsilon\to0}\frac{f(t+\varepsilon t^{1-\theta})-f(t)}{\varepsilon},\ f:(0,\infty)\to\mathbb{R}. \tag{2}$$

Some of the features of conformable fractional derivative as follows [38].

$a)\ {}_tD^\theta t^\alpha=\alpha t^{\alpha-\theta},\ \forall\theta\in\mathbb{R},$

$b)\ {}_tD^\theta(\mathrm{fg})=f\,{}_tD^\theta g+g\,{}_tD^\theta f,$

$c)\ {}_tD^\theta(\mathrm{fog})=t^{1-\theta}g'(t)f'(g(t)),$

$d)\ {}_tD^\theta(\frac{f}{g})=\frac{g\,{}_tD^\theta f-f\,{}_tD^\theta g}{g^2}.$

- Suppose the traveling wave variable:

$$\varepsilon(\sigma,t)=u(\phi),\phi=\sigma-v\frac{t^\theta}{\theta}. \tag{3}$$

Then, from Eq. (5.3), Eq. (5.1) is turn to an ordinary differential equation for

$$\omega u''(\phi)+\mathrm{v}u'(\phi)-\chi u(\phi)^2=0. \tag{4}$$

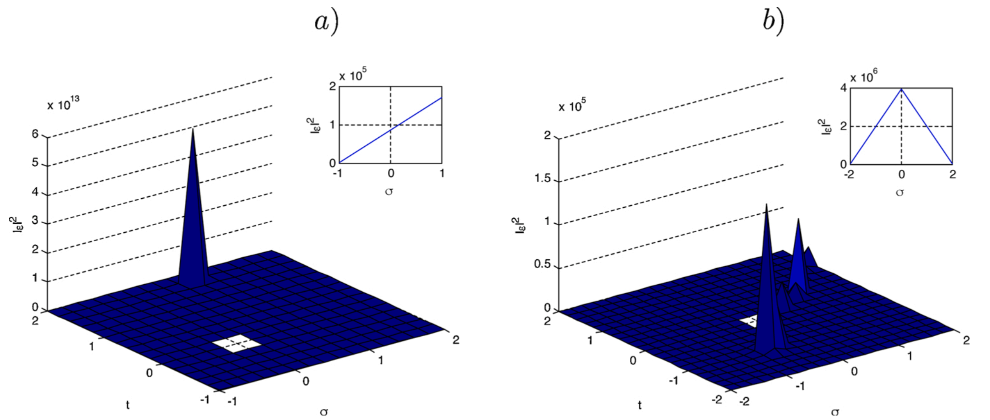

Fig. 4. The 3D graphics for $\varepsilon(\sigma,t)$ analytical solutions of the Eq. (5.1) for $\omega=1,\chi=2,v=1,\theta=0.6$. (a) for solution (5.11), (b) for solution (5.12).

Consider the solutions of Eq. (5.4) can write as a series expansion solution as follows,

$$u(\phi)=A_0+A_1G(\phi)+A_2G^{-1}(\phi)+B_1G(\phi)^2+B_2G^{-2}(\phi), \tag{5}$$

where A_0,A_1,A_2,B_1,B_2 are functions to be determined later and $G(\phi)$ satisfies the fractional Riccati equation as follows:

$$G^{'}(\phi)=\xi+G^2(\phi), \tag{6}$$

where ξ is an arbitrary constants.

•N is obtained with the aid of balance between the highest order derivatives and the nonlinear terms in Eq.(5.4).

A few special solutions of Eq. (5.5) are given by;

1) When $\xi<0$,

$$\begin{aligned} G_1(\phi)= & -\sqrt{-\xi}tanh(\sqrt{-\xi}\phi), \\ G_2(\phi)= & -\sqrt{-\xi}coth(\sqrt{-\xi}\phi), \end{aligned} \tag{7}$$

2) When $\xi>0$,

$$\begin{aligned} G_3(\phi)= & \sqrt{\xi}tan(\sqrt{\xi}\phi), \\ G_4(\phi)= & \sqrt{\xi}cot(\sqrt{\xi}\phi), \end{aligned} \tag{8}$$

3) When $\xi=0$, ρ = const.,

$$G_5(\phi)=-\frac{1}{\phi+\rho}. \tag{9}$$

Now, replacing (5.5) and (5.6) into (5.4), by equating the all coefficients of $G(\phi)$, we can solve equations. Then we obtain the following some functions:

$$\begin{aligned} A_0= & -\frac{v^2}{50\omega\chi}+\frac{4\omega\sigma}{\chi},\ A_1=\frac{6v}{5\chi},\ A_2=\frac{v^3+100v\omega^2\sigma}{250\omega^2\chi}, \\ B_1= & \frac{6\omega}{\chi},\ B_2=-\frac{7v^4+2000\omega^4\sigma^2}{5000\omega^3\chi}. \end{aligned} \tag{10}$$

Then by using $\xi=1, G(\phi)=\sqrt{\xi}tan(\sqrt{\xi}\phi)$, we obtain

$$\begin{aligned} u(\phi) = & -\frac{v^2}{50\omega\chi}+\frac{4\omega\sigma}{\chi}+\frac{6v}{5\chi}\sqrt{\xi}tan(\sqrt{\xi}\phi)+\frac{v^3+100v\omega^2\sigma}{250\omega^2\chi}(\sqrt{\xi}tan(\sqrt{\xi}\phi))^{-1} \\ & +\frac{6\omega}{\chi}(\sqrt{\xi}tan(\sqrt{\xi}\phi))^2-\frac{7v^4+2000\omega^4\sigma^2}{5000\omega^3\chi}(\sqrt{\xi}tan(\sqrt{\xi}\phi))^{-2}. \end{aligned}$$

From here,

$$\begin{aligned} \varepsilon(\sigma,t)= & -\frac{v^2}{50\omega\chi}+\frac{4\omega\sigma}{\chi}+\frac{6v}{5\chi}\sqrt{\xi}tan(\sqrt{\xi}(\sigma-v\frac{t^\theta}{\theta}))+\frac{v^3+100v\omega^2\sigma}{250\omega^2\chi}(\sqrt{\xi}tan(\sigma-v\frac{t^\theta}{\theta}))^{-1} \\ & +\frac{6\omega}{\chi}(\sqrt{\xi}tan(\sqrt{\xi}(\sigma-v\frac{t^\theta}{\theta})))^2-\frac{7v^4+2000\omega^4\sigma^2}{5000\omega^3\chi}(\sqrt{\xi}tan(\sqrt{\xi}(\sigma-v\frac{t^\theta}{\theta})))^{-2}. \end{aligned} \tag{11}$$

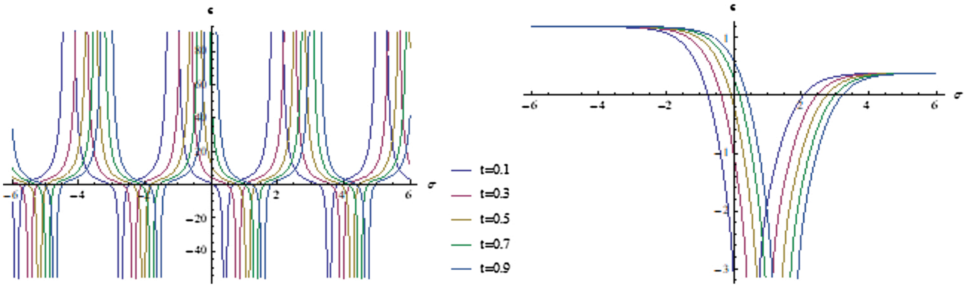

Fig. 5. The 2D graphic for $\varepsilon(\sigma,t)$ analytical solutions of Eq. (5.1) for different value of t. ($\omega=1,\chi=2,\nu=1,\theta=0.6$). (a) for solution (5.11), (b) for solution (5.12).

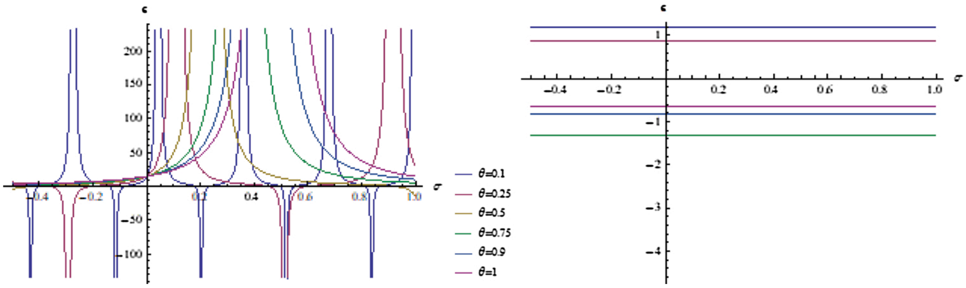

Fig. 6. The 2D graphic for $\varepsilon(\sigma,t)$ analytical solutions of the Eq. (5.1) for different value of θ. ($\omega=1,\chi=2,\nu=1,\sigma=2$). (a) for solution (5.11), (b) for solution (5.12).

By using $\xi=-1, G(\phi)=-\sqrt{-\xi}tanh(\sqrt{-\xi}\phi)$, we obtain

$$\begin{aligned} u(\phi) &= -\frac{v^2}{50\omega\chi}+\frac{4\omega\sigma}{\chi}+\frac{6v}{5\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}\phi))+\frac{v^3+100v\omega^2\sigma}{250\omega^2\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}\phi))^{-1} \\ &+\frac{6\omega}{\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}\phi))^2-\frac{7v^4+2000\omega^4\sigma^2}{5000\omega^3\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}\phi))^{-2}, \end{aligned}$$

From here,

$$\begin{aligned} \varepsilon(\sigma,t) &= -\frac{v^2}{50\omega\chi}+\frac{4\omega\sigma}{\chi}+\frac{6v}{5\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}(\sigma-v\frac{t^\theta}{\theta})))+\frac{v^3+100v\omega^2\sigma}{250\omega^2\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}(\sigma-v\frac{t^\theta}{\theta})))^{-1} \\ &+\frac{6\omega}{\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}(\sigma-v\frac{t^\theta}{\theta})))^2-\frac{7v^4+2000\omega^4\sigma^2}{5000\omega^3\chi}(-\sqrt{-\xi}tanh(\sqrt{-\xi}(\sigma-v\frac{t^\theta}{\theta})))^{-2}. \end{aligned} \quad (12)$$

Figs. 4–6

6. Conclusion

Flows theory an important part of geometric optic, optical physics, and magnetic motion, [41–51]. In our paper, we obtain some optical conditions of spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic Lorentz flux by using directional spherical fields. Moreover, we determine spherical magnetic Lorentz flux for spherical vector fields. Also, we give new construction for spherical curvatures of spherical $\mathbb{S}\boldsymbol{\alpha}$-magnetic particle flows by considering Heisenberg spherical Landau Lifshitz model. Finally, magnetic flux surface is demonstrated in a static and uniform magnetic surface by using the analytical and numerical results with Heisenberg spherical Landau Lifshitz model.

In our future work beneath this kind of idea, we offer to review spherical magnetic Lorentz flux of $\mathbb{S}$t-magnetic particles in de Sitter space. Another purpose of the future studies will be explore the unified formulations of the systems composed of arbitrary dyons, magnetic and electric charges of manifolds with magnetic flux lines.

Declaration of Competing Interest

The authors report no declarations of interest.

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