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New quasi uniformly accelerated motion with hidden quasi momentum

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ABSTRACT

We firstly characterize uniformly quasi accelerated motion (UQAM) with Fermi Walker derivation. We obtain new energy of quasi tangent magnetic biharmonic particle and Lorentz field. Thus, we give new relationship between Fermi Walker parallelism and uniformly quasi accelerated motion in Heisenberg group. Moreover, we give applied geometric characterization for a uniformly quasi accelerated motion of velocity magnetic biharmonic particles in Heisenberg space.

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1. Introduction

Electromagnetic energy incline to be a fraction distracted. Lorentz energy systems are elastic sequences of fluids, solids and gases, are full of interchange, modeling offsets, and are extremely determined by thermal and optical divergences. These frameworks are too plentiful, contain biomaterials, polymers, colloidal terminations, liquids, foams, and crystallizes, and have an immense scope of energy technologies. Proceeding these technologies importunes new optical application and depiction of complexities of these ultimate energy flux systems, [1,4–7,20,21].

Magnetic energy connections are largely produced as magnetic flux components seeing diverse descriptions of machines like potential generators, turbines, and activators. Magnetic flux is regularly embittered simultaneously operation of these structures. Melting is a consequence of extermination of electromagnetic energy at elastic magnetic phases. Thus, progression in potential of

elastic spherical magnetic surfaces is significant to increase electromagnetic energy. Also, protecting electrical energy improves at preserving ordinary reserves and temperatures [22].

Differentiable map $\phi : N \rightarrow M$ is defined biharmonic if ϕ is a critical point:

$$E_2(\phi) = \int_N \frac{1}{2} |\mathcal{T}(\phi)|^2 dv_h. \quad (1.1)$$

Here $\mathcal{T}(\phi) := \text{tr} \nabla^\phi d\phi$ is tension, [10]. Also, bitension field of ϕ is described by means of

$$\mathcal{T}_2(\phi) = -\Delta_\phi \mathcal{T}(\phi) + \text{tr} R(\mathcal{T}(\phi), d\phi) d\phi. \quad (1.2)$$

Biharmonic particles also play an essential aspect in applied geometry. Thus, some experts have characterized geometric biharmonic particles and surface equations [23,24]. On the other hand, energy approach has been collected by some constructions, [2,3,9–15].

The wave propagation of surface is influenced by the lashing procedure of the wave solutions which types of waves are incorporate in lakes, rivers, beaches and oceans, and the processes that cause them can be effective in ocean engineering. Recently, some

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basic research in ocean engineering and science have been done such as the (G'/G,1/G)-expansion method applied to the space-time fractional (2+1)- dimensional dispersive longwave and approximate long water wave equations [25], the linear and nonlinear time-fractional regularized long wave equations via the residual power series method [26], the time fractional Klein-Gordon equation by using the natural transform decomposition and iterative Shehu transform methods [27], the local meshless method applied to the fractional-order Korteweg-de Vries and Burgers' equations [28], the nonlinear water wave equations using the invariant subspaces method [29].

The construction of our article is as follows: Firstly, we characterize uniformly quasi accelerated motion (UQAM) with Fermi Walker derivation in Heisenberg space. We obtain new energy of quasi velocity magnetic biharmonic particles and Lorentz fields. Also, we construct a new relationship between Fermi Walker parallel transportation and uniformly quasi accelerated motion in the Heisenberg space.

2. Preliminaries

The Heisenberg metric [17] is given by

$$g_h = dx^2 + dy^2 + (dz - xdy)^2. \quad (2.1)$$

Basis for Heisenberg space [17] is presented by

$$\mathbf{p}_1 = \frac{\partial}{\partial x}, \mathbf{p}_2 = \frac{\partial}{\partial y} + x \frac{\partial}{\partial z}, \mathbf{p}_3 = \frac{\partial}{\partial z}. \quad (2.2)$$

Let α be smooth particle in the Heisenberg space. Additionally, $\mathbf{e}_{(0)}^\pi$, $\mathbf{e}_{(1)}^\pi$, and $\mathbf{e}_{(2)}^\pi$ be tangent, principal normal, and secondary normal vector field, respectively [8]. Thus, Serret-Frenet formulas are

$$\frac{d\alpha}{ds} = \mathbf{e}_{(0)}^\pi,$$

$$\begin{bmatrix} \nabla_{\mathbf{e}_{(0)}^\pi} \mathbf{e}_{(0)}^\pi \\ \nabla_{\mathbf{e}_{(0)}^\pi} \mathbf{e}_{(1)}^\pi \\ \nabla_{\mathbf{e}_{(0)}^\pi} \mathbf{e}_{(2)}^\pi \end{bmatrix} = \begin{bmatrix} & \kappa & \\ -\kappa & & \tau \\ & -\tau & \end{bmatrix} \begin{bmatrix} \mathbf{e}_{(0)}^\pi \\ \mathbf{e}_{(1)}^\pi \\ \mathbf{e}_{(2)}^\pi \end{bmatrix}, \quad (2.3)$$

where κ and τ are curvatures of the particle.

Quasi frame of α is provided by Dede et al. [8],

$$\mathbf{e}_{(0)}^\pi = \mathbf{e}_{(0)}^\pi, \mathbf{e}_{(1)}^\pi = \frac{\mathbf{e}_{(0)}^\pi \times \mathbf{k}}{\|\mathbf{e}_{(0)}^\pi \times \mathbf{k}\|}, \mathbf{e}_{(2)}^\pi = \mathbf{e}_{(0)}^\pi \wedge \mathbf{e}_{(1)}^\pi, \quad (2.4)$$

where projection field is $\mathbf{k} = (0, 0, 1)$.

Thus, quasi frame equations are given by

$$\begin{aligned} \nabla_{\mathbf{e}_{(0)}^\pi} \mathbf{e}_{(0)}^\pi &= \kappa_1 \mathbf{e}_{(1)}^\pi + \kappa_2 \mathbf{e}_{(2)}^\pi, \\ \nabla_{\mathbf{e}_{(0)}^\pi} \mathbf{e}_{(1)}^\pi &= -\kappa_1 \mathbf{e}_{(0)}^\pi + \kappa_3 \mathbf{e}_{(2)}^\pi, \\ \nabla_{\mathbf{e}_{(0)}^\pi} \mathbf{e}_{(2)}^\pi &= -\kappa_2 \mathbf{e}_{(0)}^\pi - \kappa_3 \mathbf{e}_{(1)}^\pi, \end{aligned} \quad (2.5)$$

where the angle ψ is between $\mathbf{e}_{(1)}^\pi$ and $\mathbf{e}_{(2)}^\pi$.

Also, quasi curvatures are obtained by

$$\kappa_1 = \kappa \cos \psi, \quad \kappa_2 = -\kappa \sin \psi, \quad \kappa_3 = \psi' + \tau, \quad (2.6)$$

and

$$\mathbf{e}_{(1)}^\pi \times \mathbf{e}_{(2)}^\pi = \mathbf{e}_{(0)}^\pi, \quad \mathbf{e}_{(2)}^\pi \times \mathbf{e}_{(0)}^\pi = \mathbf{e}_{(1)}^\pi, \quad \mathbf{e}_{(0)}^\pi \times \mathbf{e}_{(1)}^\pi = \mathbf{e}_{(2)}^\pi. \quad (2.7)$$

3. Velocity quasi magnetic biharmonic particles with quasi frame

Definition 3.1. Velocity quasi magnetic particle is defined by Lorentz equation:

$$\nabla_s \mathbf{e}_{(0)}^\pi = \phi(\mathbf{e}_{(0)}^\pi) = \mathcal{B} \times \mathbf{e}_{(0)}^\pi. \quad (3.1)$$

Proposition 3.2. Lorentz forces of $\mathbf{e}_{(0)}^\pi$, $\mathbf{e}_{(1)}^\pi$, $\mathbf{e}_{(2)}^\pi$ are presented

$$\begin{aligned} \phi(\mathbf{e}_{(0)}^\pi) &= (\kappa_1 \cos[\chi_1 s + \chi_2] + \kappa_2 \cos \varphi \sin[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + (\kappa_2 \cos \varphi \cos[\chi_1 s + \chi_2] - \kappa_1 \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - \kappa_2 \sin \varphi \mathbf{p}_3, \\ \phi(\mathbf{e}_{(1)}^\pi) &= (\varpi \cos \varphi \sin[\chi_1 s + \chi_2] - \kappa_1 \sin \varphi \sin[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + (\varpi \cos \varphi \cos[\chi_1 s + \chi_2] - \kappa_1 \sin \varphi \cos[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - (\kappa_1 \cos \varphi + \varpi \sin \varphi) \mathbf{p}_3, \\ \phi(\mathbf{e}_{(2)}^\pi) &= -(\kappa_2 \sin \varphi \sin[\chi_1 s + \chi_2] + \varpi \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + (\varpi \sin[\chi_1 s + \chi_2] - \kappa_2 \sin \varphi \cos[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - \kappa_2 \cos \varphi \mathbf{p}_3. \end{aligned} \quad (3.2)$$

Proof. It is obvious by using Definition 3.1. \square

Proposition 3.3. Magnetic vector field \mathcal{B} of a normal magnetic biharmonic particle is given by

$$\begin{aligned} \mathcal{B} &= (\varpi \sin \varphi \sin[\chi_1 s + \chi_2] - \kappa_2 \cos[\chi_1 s + \chi_2] \\ &\quad + \kappa_1 \cos \varphi \sin[\chi_1 s + \chi_2]) \mathbf{p}_1 + (\kappa_2 \sin[\chi_1 s + \chi_2] \\ &\quad + \varpi \sin \varphi \cos[\chi_1 s + \chi_2] + \kappa_1 \cos \varphi \cos[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad + (\varpi \cos \varphi - \kappa_1 \sin \varphi) \mathbf{p}_3. \end{aligned} \quad (3.3)$$

Proof. It is obvious by using Definition 3.1. \square

4. Uniformly quasi accelerated motion

In this part, we obtain uniformly quasi accelerated motion of velocity quasi magnetic biharmonic particles in Heisenberg space. Thus, we get following definition of uniformly new quasi accelerated theory [14–19].

Definition 4.1. ∇_s^{FVW} is defined by

$$\nabla_s^{FVW} \mathcal{R} = \nabla_s \mathcal{R} - h(\mathbf{T}, \mathcal{R}) \nabla_s \mathbf{T} + h(\nabla_s \mathbf{T}, \mathcal{R}) \mathbf{T}, \quad (4.1)$$

where \mathcal{R} is a field 20.

Definition 4.2. The particle \mathbf{X} is UQAM iff

$$\nabla_s^{FVW} (\nabla_s \mathbf{X}) = 0. \quad (4.2)$$

Theorem 4.3. Lorentz forces $\phi(\mathbf{e}_{(0)}^\pi)$, $\phi(\mathbf{e}_{(1)}^\pi)$, $\phi(\mathbf{e}_{(2)}^\pi)$ are Fermi Walker parallel iff

$$\begin{aligned} &((\kappa_2' + \kappa_1 \kappa_3) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + (\kappa_1' - \kappa_2 \kappa_3) \cos[\chi_1 s + \chi_2]) = 0, \\ &((\kappa_2' + \kappa_1 \kappa_3) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1' - \kappa_2 \kappa_3) \sin[\chi_1 s + \chi_2]) = 0, \\ &-(\kappa_2' + \kappa_1 \kappa_3) \sin \varphi = 0, \\ &(\varpi' \cos \varphi \sin[\chi_1 s + \chi_2] - \kappa_1' \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - \kappa_3 \varpi \cos[\chi_1 s + \chi_2]) = 0, \\ &(\varpi' \cos \varphi \cos[\chi_1 s + \chi_2] - \kappa_1' \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad + \kappa_3 \varpi \sin[\chi_1 s + \chi_2]) = 0, \\ &(\varpi' \sin \varphi + \kappa_1' \cos \varphi) = 0, \end{aligned} \quad (4.3)$$

$$\begin{aligned} &(\varpi \kappa_3 \cos \varphi \sin[\chi_1 s + \chi_2] + \kappa_2' \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + \varpi' \cos[\chi_1 s + \chi_2]) = 0, \\ &(\varpi \kappa_3 \cos \varphi \cos[\chi_1 s + \chi_2] + \kappa_2' \sin \varphi \cos[\chi_1 s + \chi_2] \end{aligned}$$

$$\begin{aligned} -\varpi' \sin[\chi_1 s + \chi_2] &= 0, \\ (\varpi \kappa_3 \sin \varphi - \kappa_2' \cos \varphi) &= 0. \end{aligned}$$

Proof. Fermi Walker derivatives of $\phi(\mathbf{e}_{(0)}^\pi)$, $\phi(\mathbf{e}_{(1)}^\pi)$, $\phi(\mathbf{e}_{(2)}^\pi)$ are

$$\begin{aligned} \nabla_s^{\mathcal{F}W} \phi(\mathbf{e}_{(0)}^\pi) &= ((\kappa_2' + \kappa_1 \kappa_3) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + (\kappa_1' - \kappa_2 \kappa_3) \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + ((\kappa_2' + \kappa_1 \kappa_3) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1' - \kappa_2 \kappa_3) \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - (\kappa_2' + \kappa_1 \kappa_3) \sin \varphi \mathbf{p}_3, \end{aligned}$$

$$\begin{aligned} \nabla_s^{\mathcal{F}W} \phi(\mathbf{e}_{(1)}^\pi) &= (\varpi' \cos \varphi \sin[\chi_1 s + \chi_2] - \kappa_1' \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - \kappa_3 \varpi \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 + (\varpi' \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - \kappa_1' \sin \varphi \cos[\chi_1 s + \chi_2] + \kappa_3 \varpi \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - (\varpi' \sin \varphi + \kappa_1' \cos \varphi) \mathbf{p}_3, \end{aligned} \quad (4.4)$$

$$\begin{aligned} \nabla_s^{\mathcal{F}W} \phi(\mathbf{e}_{(2)}^\pi) &= -(\varpi \kappa_3 \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + \kappa_2' \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + \varpi' \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 - (\varpi \kappa_3 \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad + \kappa_2' \sin \varphi \cos[\chi_1 s + \chi_2] - \varpi' \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad + (\varpi \kappa_3 \sin \varphi - \kappa_2' \cos \varphi) \mathbf{p}_3. \end{aligned}$$

So, by using the above equation system and considering Definition 4.1, some algebraic calculus shows that theorem. \square

Uniformly Quasi Accelerated Motion

In this subsection, we will characterize uniformly quasi accelerated motion (UQAM) in Heisenberg space.

Theorem 4.4. The $\phi(\mathbf{e}_{(0)}^\pi)$ obeys a uniformly quasi accelerated motion iff

$$\begin{aligned} &(((\kappa_2' + \kappa_1 \kappa_3)' + \kappa_3(\kappa_1' - \kappa_2 \kappa_3)) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1^2 + \kappa_2^2)') \sin \varphi \sin[\chi_1 s + \chi_2] + ((\kappa_1' - \kappa_2 \kappa_3)' \\ &\quad - \kappa_3(\kappa_2' + \kappa_1 \kappa_3)) \cos[\chi_1 s + \chi_2]) = 0, \\ &(((\kappa_2' + \kappa_1 \kappa_3)' + \kappa_3(\kappa_1' - \kappa_2 \kappa_3)) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1^2 + \kappa_2^2)') \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1' - \kappa_2 \kappa_3)' - \kappa_3(\kappa_2' + \kappa_1 \kappa_3)) \sin[\chi_1 s + \chi_2]) = 0, \\ &(((\kappa_2' + \kappa_1 \kappa_3)' + \kappa_3(\kappa_1' - \kappa_2 \kappa_3)) \sin \varphi + ((\kappa_1^2 + \kappa_2^2)') \cos \varphi) = 0. \end{aligned} \quad (4.5)$$

Proof. From Fermi Walker derivative, we get

$$\begin{aligned} \nabla_s \phi(\mathbf{e}_{(0)}^\pi) &= ((\kappa_2' + \kappa_1 \kappa_3) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1^2 + \kappa_2^2) \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + (\kappa_1' - \kappa_2 \kappa_3) \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + ((\kappa_2' + \kappa_1 \kappa_3) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1^2 + \kappa_2^2) \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1' - \kappa_2 \kappa_3) \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - ((\kappa_2' + \kappa_1 \kappa_3) \sin \varphi + (\kappa_1^2 + \kappa_2^2) \cos \varphi) \mathbf{p}_3. \end{aligned} \quad (4.6)$$

It provides us the following equation

$$\begin{aligned} \nabla_s^{\mathcal{F}W} \nabla_s \phi(\mathbf{e}_{(0)}^\pi) &= (((\kappa_2' + \kappa_1 \kappa_3)' + \kappa_3(\kappa_1' - \kappa_2 \kappa_3)) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1^2 + \kappa_2^2)') \sin \varphi \sin[\chi_1 s + \chi_2] + ((\kappa_1' - \kappa_2 \kappa_3)' \\ &\quad - \kappa_3(\kappa_2' + \kappa_1 \kappa_3)) \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 + (((\kappa_2' + \kappa_1 \kappa_3)' \\ &\quad + \kappa_3(\kappa_1' - \kappa_2 \kappa_3)) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1^2 + \kappa_2^2)') \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1' - \kappa_2 \kappa_3)' - \kappa_3(\kappa_2' + \kappa_1 \kappa_3)) \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - (((\kappa_2' + \kappa_1 \kappa_3)' + \kappa_3(\kappa_1' - \kappa_2 \kappa_3)) \sin \varphi \\ &\quad + ((\kappa_1^2 + \kappa_2^2)') \cos \varphi) \mathbf{p}_3. \end{aligned} \quad (4.7)$$

And, by using above equation, we can easily theorem. \square

Theorem 4.5. The $\phi(\mathbf{e}_{(1)}^\pi)$ obeys a uniformly quasi accelerated motion iff

$$\begin{aligned} &(((\varpi' - \kappa_1 \kappa_2)' - (\kappa_1^2 + \kappa_3 \varpi) \kappa_3) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1' + \kappa_2 \varpi)') \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1^2 + \kappa_3 \varpi)' + \kappa_3(\varpi' - \kappa_1 \kappa_2)) \cos[\chi_1 s + \chi_2]) = 0, \\ &(((\varpi' - \kappa_1 \kappa_2)' - (\kappa_1^2 + \kappa_3 \varpi) \kappa_3) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1' + \kappa_2 \varpi)') \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad + ((\kappa_1^2 + \kappa_3 \varpi)' + \kappa_3(\varpi' - \kappa_1 \kappa_2)) \sin[\chi_1 s + \chi_2]) = 0, \\ &(((\varpi' - \kappa_1 \kappa_2)' - (\kappa_1^2 + \kappa_3 \varpi) \kappa_3) \sin \varphi \\ &\quad + ((\kappa_1' + \kappa_2 \varpi)') \cos \varphi) = 0. \end{aligned} \quad (4.8)$$

Proof. From $\phi(\mathbf{e}_{(1)}^\pi)$, we get

$$\begin{aligned} \nabla_s \phi(\mathbf{e}_{(1)}^\pi) &= ((\varpi' - \kappa_1 \kappa_2) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1' + \kappa_2 \varpi) \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1^2 + \kappa_3 \varpi) \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + ((\varpi' - \kappa_1 \kappa_2) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - (\kappa_1' + \kappa_2 \varpi) \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad + (\kappa_1^2 + \kappa_3 \varpi) \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - ((\varpi' - \kappa_1 \kappa_2) \sin \varphi + (\kappa_1' + \kappa_2 \varpi) \cos \varphi) \mathbf{p}_3. \end{aligned} \quad (4.9)$$

Since

$$\begin{aligned} \nabla_s^{\mathcal{F}W} \nabla_s \phi(\mathbf{e}_{(1)}^\pi) &= (((\varpi' - \kappa_1 \kappa_2)' - (\kappa_1^2 + \kappa_3 \varpi) \kappa_3) \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1' + \kappa_2 \varpi)') \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1^2 + \kappa_3 \varpi)' + \kappa_3(\varpi' - \kappa_1 \kappa_2)) \cos[\chi_1 s + \chi_2]) \mathbf{p}_1 \\ &\quad + (((\varpi' - \kappa_1 \kappa_2)' - (\kappa_1^2 + \kappa_3 \varpi) \kappa_3) \cos \varphi \cos[\chi_1 s + \chi_2] \\ &\quad - ((\kappa_1' + \kappa_2 \varpi)') \sin \varphi \cos[\chi_1 s + \chi_2] \\ &\quad + ((\kappa_1^2 + \kappa_3 \varpi)' + \kappa_3(\varpi' - \kappa_1 \kappa_2)) \sin[\chi_1 s + \chi_2]) \mathbf{p}_2 \\ &\quad - (((\varpi' - \kappa_1 \kappa_2)' - (\kappa_1^2 + \kappa_3 \varpi) \kappa_3) \sin \varphi \\ &\quad + ((\kappa_1' + \kappa_2 \varpi)') \cos \varphi) \mathbf{p}_3. \end{aligned} \quad (4.10)$$

\square

Theorem 4.6. The $\phi(\mathbf{e}_{(2)}^\pi)$ obeys a uniformly quasi accelerated motion iff

$$\begin{aligned} &((-\varpi' + \kappa_2 \kappa_1) \kappa_3 - (\kappa_2^2 + \varpi \kappa_3)') \cos \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + (\varpi \kappa_1 - \kappa_2') \sin \varphi \sin[\chi_1 s + \chi_2] \\ &\quad + (-\varpi' + \kappa_2 \kappa_1)' + \kappa_3(\kappa_2^2 + \varpi \kappa_3)) \cos[\chi_1 s + \chi_2]) = 0, \\ &((-\varpi' + \kappa_2 \kappa_1) \kappa_3 - (\kappa_2^2 + \varpi \kappa_3)') \cos \varphi \cos[\chi_1 s + \chi_2] \end{aligned}$$

$$\begin{aligned}
 & +(\varpi \kappa_1 - \kappa_2') \sin \varphi \cos [\chi_1 s + \chi_2] \\
 & -(-(\varpi' + \kappa_2 \kappa_1)' + \kappa_3(\kappa_2^2 + \varpi \kappa_3)) \sin [\chi_1 s + \chi_2] = 0, \\
 & +((\varpi \kappa_1 - \kappa_2') \cos \varphi - (- (\varpi' + \kappa_2 \kappa_1) \kappa_3 \\
 & - (\kappa_2^2 + \varpi \kappa_3)') \sin \varphi) = 0.
 \end{aligned} \tag{4.11}$$

Proof. Similarly, we have

$$\begin{aligned}
 \nabla_s \phi(\mathbf{e}_{(2)}^\pi) & = ((\varpi \kappa_1 - \kappa_2') \sin \varphi \sin [\chi_1 s + \chi_2] \\
 & - (\varpi' + \kappa_2 \kappa_1) \cos [\chi_1 s + \chi_2] \\
 & - (\kappa_2^2 + \varpi \kappa_3) \cos \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 \\
 & + ((\varpi \kappa_1 - \kappa_2') \sin \varphi \cos [\chi_1 s + \chi_2] \\
 & + (\varpi' + \kappa_2 \kappa_1) \sin [\chi_1 s + \chi_2] \\
 & - (\kappa_2^2 + \varpi \kappa_3) \cos \varphi \cos [\chi_1 s + \chi_2]) \mathbf{p}_2 \\
 & + ((\varpi \kappa_1 - \kappa_2') \cos \varphi + (\kappa_2^2 + \varpi \kappa_3) \sin \varphi) \mathbf{p}_3.
 \end{aligned} \tag{4.12}$$

Thus, we put

$$\begin{aligned}
 \nabla_s^{FW} \nabla_s \phi(\mathbf{e}_{(2)}^\pi) & = ((-\varpi' + \kappa_2 \kappa_1) \kappa_3 - (\kappa_2^2 + \varpi \kappa_3)') \\
 & \cos \varphi \sin [\chi_1 s + \chi_2] \\
 & + (\varpi \kappa_1 - \kappa_2') \sin \varphi \sin [\chi_1 s + \chi_2] \\
 & + (- (\varpi' + \kappa_2 \kappa_1)' + \kappa_3(\kappa_2^2 + \varpi \kappa_3)) \\
 & \cos [\chi_1 s + \chi_2]) \mathbf{p}_1 + ((-\varpi' + \kappa_2 \kappa_1) \kappa_3 \\
 & - (\kappa_2^2 + \varpi \kappa_3)') \cos \varphi \cos [\chi_1 s + \chi_2] \\
 & + (\varpi \kappa_1 - \kappa_2') \sin \varphi \cos [\chi_1 s + \chi_2] \\
 & - (- (\varpi' + \kappa_2 \kappa_1)' + \kappa_3(\kappa_2^2 + \varpi \kappa_3)) \\
 & \sin [\chi_1 s + \chi_2]) \mathbf{p}_2 + ((\varpi \kappa_1 - \kappa_2') \cos \varphi \\
 & - (- (\varpi' + \kappa_2 \kappa_1) \kappa_3 - (\kappa_2^2 + \varpi \kappa_3)') \sin \varphi) \mathbf{p}_3.
 \end{aligned} \tag{4.13}$$

This completes the proof. \square

5. Energy quasi flux density

In cold plasma, we have

$$\mathbf{F} = q(\mathcal{E} + \mathbf{v} \times \mathcal{B}), \quad \frac{m}{q} = \sigma, \tag{5.1}$$

where \mathbf{v} is a velocity of electrons, m is a mass, q is an electric charge, \mathcal{B} is a magnetic field and \mathcal{E} is an electric field [18,19].

From force equation, electric field is presented by

$$\begin{aligned}
 \mathcal{E} & = ((\sigma \kappa_2 + \kappa_1) \cos \varphi \sin [\chi_1 s + \chi_2] + (\sigma \kappa_1 + \kappa_2) \cos [\chi_1 s + \chi_2]) \mathbf{p}_1 \\
 & + ((\sigma \kappa_2 + \kappa_1) \cos \varphi \cos [\chi_1 s + \chi_2] \\
 & + - (\sigma \kappa_1 \kappa_2) \sin [\chi_1 s + \chi_2]) \mathbf{p}_2 - (\sigma \kappa_2 + \kappa_1) \sin \varphi \mathbf{p}_3.
 \end{aligned} \tag{5.2}$$

Theorem 5.1. The \mathcal{E} obeys a uniformly quasi accelerated motion iff

$$\begin{aligned}
 & (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))' + \kappa_3((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 \\
 & + \kappa_1))) \cos \varphi \sin [\chi_1 s + \chi_2] + (((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 + \kappa_1))' \\
 & - \kappa_3((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))) \cos [\chi_1 s + \chi_2] \\
 & - ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1))' \sin \varphi \sin [\chi_1 s + \chi_2]) = 0, \\
 & (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))' + \kappa_3((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 \\
 & + \kappa_1))) \cos \varphi \cos [\chi_1 s + \chi_2] - ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 \\
 & + \kappa_1))' \sin \varphi \cos [\chi_1 s + \chi_2] - (((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 + \kappa_1))' \\
 & - \kappa_3((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))) \sin [\chi_1 s + \chi_2]) = 0, \\
 & (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))' + \kappa_3((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2
 \end{aligned}$$

$$\begin{aligned}
 & + \kappa_1))) \sin \varphi + ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1))' \cos \varphi) = 0.
 \end{aligned} \tag{5.3}$$

Proof. By the formula of the electric field, we have

$$\begin{aligned}
 \nabla_s \mathcal{E} & = (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2)) \cos \varphi \sin [\chi_1 s + \chi_2] \\
 & - (\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \sin \varphi \sin [\chi_1 s + \chi_2] \\
 & + ((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 + \kappa_1)) \cos [\chi_1 s + \chi_2]) \mathbf{p}_1 \\
 & + (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2)) \cos \varphi \cos [\chi_1 s + \chi_2] \\
 & - (\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \sin \varphi \cos [\chi_1 s + \chi_2] \\
 & - ((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 + \kappa_1)) \sin [\chi_1 s + \chi_2]) \mathbf{p}_2 \\
 & - (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2)) \sin \varphi \\
 & + (\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \cos \varphi) \mathbf{p}_3.
 \end{aligned} \tag{5.4}$$

So, we get

$$\begin{aligned}
 \nabla_s^{FW} \nabla_s \mathcal{E} & = (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))' + \kappa_3((\sigma \kappa_1 + \kappa_2)' \\
 & - \kappa_3(\sigma \kappa_2 + \kappa_1))) \cos \varphi \sin [\chi_1 s + \chi_2] + (((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 \\
 & + \kappa_1))' - \kappa_3((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))) \cos [\chi_1 s + \chi_2] \\
 & - ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1))' \sin \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 \\
 & + (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))' + \kappa_3((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 \\
 & + \kappa_1))) \cos \varphi \cos [\chi_1 s + \chi_2] - ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 \\
 & + \kappa_1))' \sin \varphi \cos [\chi_1 s + \chi_2] - (((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 + \kappa_1))' \\
 & - \kappa_3((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))) \sin [\chi_1 s + \chi_2]) \mathbf{p}_2 \\
 & - (((\sigma \kappa_2 + \kappa_1)' + \kappa_3(\sigma \kappa_1 + \kappa_2))' + \kappa_3((\sigma \kappa_1 + \kappa_2)' - \kappa_3(\sigma \kappa_2 \\
 & + \kappa_1))) \sin \varphi + ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1))' \cos \varphi) \mathbf{p}_3.
 \end{aligned} \tag{5.5}$$

\square

Theorem 5.2. The \mathcal{B} obeys a uniformly quasi accelerated motion iff

$$\begin{aligned}
 & (\varpi'' \sin \varphi \sin [\chi_1 s + \chi_2] + ((\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2') - \kappa_3(\varpi \kappa_2 \\
 & - \kappa_2 \kappa_3 + \kappa_1')) \cos [\chi_1 s + \chi_2] + ((\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \\
 & + \kappa_3(\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2')) \cos \varphi \sin [\chi_1 s + \chi_2]) = 0, \\
 & (\varpi'' \sin \varphi \cos [\chi_1 s + \chi_2] - ((\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2') - \kappa_3(\varpi \kappa_2 \\
 & - \kappa_2 \kappa_3 + \kappa_1')) \sin [\chi_1 s + \chi_2] + ((\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \\
 & + \kappa_3(\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2')) \cos \varphi \cos [\chi_1 s + \chi_2]) = 0, \\
 & (\varpi'' \cos \varphi - ((\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \\
 & + \kappa_3(\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2')) \sin \varphi) = 0.
 \end{aligned} \tag{5.6}$$

Proof. From magnetic field, we get

$$\begin{aligned}
 \nabla_s \mathcal{B} & = (\varpi' \sin \varphi \sin [\chi_1 s + \chi_2] + (\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2') \cos [\chi_1 s + \chi_2] \\
 & + (\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \cos \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 \\
 & + (\varpi' \sin \varphi \cos [\chi_1 s + \chi_2] \\
 & - (\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2') \sin [\chi_1 s + \chi_2] + (\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \\
 & \cos \varphi \cos [\chi_1 s + \chi_2]) \mathbf{p}_2 \\
 & + (\varpi' \cos \varphi - (\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \sin \varphi) \mathbf{p}_3.
 \end{aligned} \tag{5.7}$$

From Fermi Walker derivative and following equation we have above system.

$$\begin{aligned}
 \nabla_s^{FW} \nabla_s \mathcal{B} & = (\varpi'' \sin \varphi \sin [\chi_1 s + \chi_2] + ((\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2') \\
 & - \kappa_3(\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1')) \cos [\chi_1 s + \chi_2] \\
 & + ((\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1')' + \kappa_3(\varpi \kappa_1 - \kappa_1 \kappa_3 \\
 & - \kappa_2')) \cos \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 \\
 & + (\varpi'' \sin \varphi \cos [\chi_1 s + \chi_2] \\
 & - ((\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2')' - \kappa_3(\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1')) \\
 & \sin [\chi_1 s + \chi_2] + ((\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1')' \\
 & + \kappa_3(\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2')) \cos \varphi \cos [\chi_1 s + \chi_2]) \mathbf{p}_2
 \end{aligned}$$

$$\begin{aligned}
 &+(\varpi'' \cos \varphi - ((\varpi \kappa_2 - \kappa_2 \kappa_3 + \kappa_1') \\
 &+\kappa_3(\varpi \kappa_1 - \kappa_1 \kappa_3 - \kappa_2') \sin \varphi) \mathbf{p}_3.
 \end{aligned} \tag{5.8}$$

□

In terms of electromagnetic fields, we may give:

By using velocity quasi magnetic fields, we get

$$\begin{aligned}
 \phi(\mathcal{E}) = & -((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \sin \varphi \sin [\chi_1 s + \chi_2] \\
 & + \varpi(\sigma \kappa_2 + \kappa_1) \cos [\chi_1 s + \chi_2] - \varpi(\sigma \kappa_1 + \kappa_2) \\
 & \cos \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 - ((\kappa_1(\sigma \kappa_1 + \kappa_2) \\
 & + \kappa_2(\sigma \kappa_2 + \kappa_1)) \sin \varphi \cos [\chi_1 s + \chi_2] - \varpi(\sigma \kappa_2 + \kappa_1) \\
 & \sin [\chi_1 s + \chi_2] - \varpi(\sigma \kappa_1 + \kappa_2) \cos \varphi \cos [\chi_1 s + \chi_2]) \mathbf{p}_2 \\
 & - ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \cos \varphi \\
 & + \varpi(\sigma \kappa_1 + \kappa_2) \sin \varphi) \mathbf{p}_3.
 \end{aligned} \tag{5.9}$$

Thus, energy flux density is given by

$$\begin{aligned}
 S = & ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \sin \varphi \sin [\chi_1 s + \chi_2] \\
 & + \varpi(\sigma \kappa_2 + \kappa_1) \cos [\chi_1 s + \chi_2] - \varpi(\sigma \kappa_1 + \kappa_2) \\
 & \cos \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 + ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \\
 & \sin \varphi \cos [\chi_1 s + \chi_2] - \varpi(\sigma \kappa_2 + \kappa_1) \sin [\chi_1 s + \chi_2] \\
 & - \varpi(\sigma \kappa_1 + \kappa_2) \cos \varphi \cos [\chi_1 s + \chi_2]) \mathbf{p}_2 \\
 & + ((\kappa_1(\sigma \kappa_1 + \kappa_2) + \kappa_2(\sigma \kappa_2 + \kappa_1)) \cos \varphi \\
 & + \varpi(\sigma \kappa_1 + \kappa_2) \sin \varphi) \mathbf{p}_3.
 \end{aligned} \tag{5.10}$$

6. Application to potential electrical energy

In this part, we construct polar plot for time variation of potential electrical energy in the cold plasma by its electric field and energy flux density in radial direction.

- The electrical energy of the electric field in the electric field \mathcal{E}

$$\begin{aligned}
 \Pi^\mathcal{E}(\phi(\mathbf{e}_{(0)}^\pi)) = & (\cos [\chi_1 s + \chi_2] \beta_1 + \cos \varphi \sin [\chi_1 s + \chi_2] \beta_2) \\
 & ((\sigma \kappa_2 + \kappa_1) \cos \varphi \sin [\chi_1 s + \chi_2] + (\sigma \kappa_1 + \kappa_2) \\
 & \cos [\chi_1 s + \chi_2]) + (\cos \varphi \cos [\chi_1 s + \chi_2] \beta_2 \\
 & - \sin [\chi_1 s + \chi_2] \beta_1) ((\sigma \kappa_2 + \kappa_1) \cos \varphi \cos [\chi_1 s + \chi_2] \\
 & - (\sigma \kappa_1 + \kappa_2) \sin [\chi_1 s + \chi_2]) \\
 & + \sin \varphi \beta_2 (\sigma \kappa_2 + \kappa_1) \sin \varphi.
 \end{aligned} \tag{6.1}$$

Fig. 1 presents the polar modeling for quasi time variation of electrical energy of $\phi(\mathbf{e}_{(0)}^\pi)$ corresponding to quasi radial direction.

$$\begin{aligned}
 \Pi^\mathcal{E}(\phi(\mathbf{e}_{(1)}^\pi)) = & (\varpi \sin [\chi_1 s + \chi_2] \cos \varphi - \sin \varphi \beta_1 \sin [\chi_1 s + \chi_2]) \\
 & ((\sigma \kappa_2 + \kappa_1) \cos \varphi \sin [\chi_1 s + \chi_2] + (\sigma \kappa_1 + \kappa_2) \\
 & \cos [\chi_1 s + \chi_2]) + (\varpi \cos [\chi_1 s + \chi_2] \cos \varphi \\
 & - \sin \varphi \beta_1 \cos [\chi_1 s + \chi_2]) ((\sigma \kappa_2 + \kappa_1) \\
 & \cos \varphi \cos [\chi_1 s + \chi_2] - (\sigma \kappa_1 + \kappa_2) \sin [\chi_1 s + \chi_2]) \\
 & + (\beta_1 \cos \varphi + \varpi \sin \varphi) (\sigma \kappa_2 + \kappa_1) \sin \varphi.
 \end{aligned} \tag{6.2}$$

Fig. 2 presents the polar modeling for quasi time variation of electrical energy of $\phi(\mathbf{e}_{(1)}^\pi)$ corresponding to quasi radial direction.

$$\begin{aligned}
 \Pi^\mathcal{E}(\phi(\mathbf{e}_{(2)}^\pi)) = & -(\sin \varphi \beta_2 \sin [\chi_1 s + \chi_2] + \cos [\chi_1 s + \chi_2] \varpi) \\
 & ((\sigma \kappa_2 + \kappa_1) \cos \varphi \sin [\chi_1 s + \chi_2] + (\sigma \kappa_1 + \kappa_2) \\
 & \cos [\chi_1 s + \chi_2]) + (\sin [\chi_1 s + \chi_2] \varpi \\
 & - \beta_2 \sin \varphi \cos [\chi_1 s + \chi_2]) ((\sigma \kappa_2 + \kappa_1) \\
 & \cos \varphi \cos [\chi_1 s + \chi_2] - (\sigma \kappa_1 + \kappa_2) \sin [\chi_1 s + \chi_2]) \\
 & - \beta_2 \cos \varphi (\sigma \kappa_2 + \kappa_1) \sin \varphi.
 \end{aligned} \tag{6.3}$$

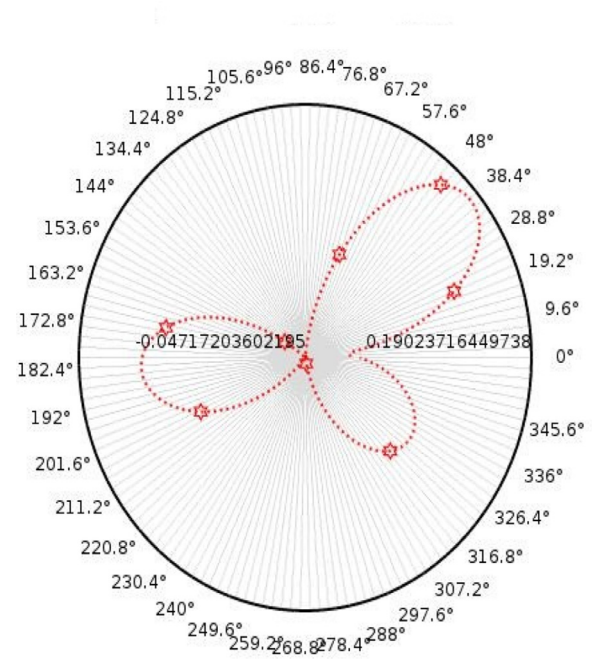


Fig. 1. Electrical energy of $\phi(\mathbf{e}_{(0)}^\pi)$ in the electric field \mathcal{E} .

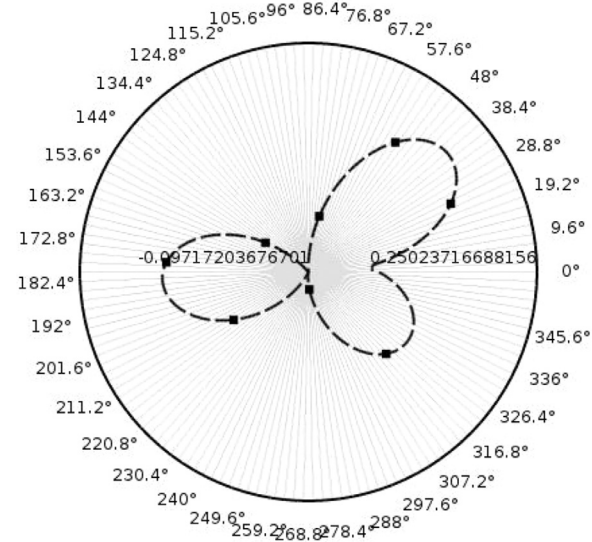


Fig. 2. Electrical energy of $\phi(\mathbf{e}_{(1)}^\pi)$ in the electric field \mathcal{E} .

Fig. 3 presents the polar modeling for quasi time variation of electrical energy of $\phi(\mathbf{e}_{(2)}^\pi)$ corresponding to quasi radial direction.

- The electrical energy of the electric field in the magnetic field \mathcal{B}

$$\begin{aligned}
 \Pi^\mathcal{E}(\mathcal{B}) = & ((\sigma \kappa_2 + \kappa_1) \cos \varphi \sin [\chi_1 s + \chi_2] + (\sigma \kappa_1 + \kappa_2) \\
 & \cos [\chi_1 s + \chi_2])^2 + ((\sigma \kappa_2 + \kappa_1) \cos \varphi \cos [\chi_1 s + \chi_2] \\
 & - (\sigma \kappa_1 + \kappa_2) \sin [\chi_1 s + \chi_2])^2 + (\sigma \kappa_2 + \kappa_1)^2 \sin^2 \varphi.
 \end{aligned} \tag{6.4}$$

Fig. 4 presents the polar modeling for quasi time variation of electrical energy of \mathcal{B} corresponding to quasi radial direction.

7. Physical and geometrical properties with conclusions

It can be noted that the energy flux density and the string flux tension for the model have not a singularity in the Heisenberg space with a quasi frame.

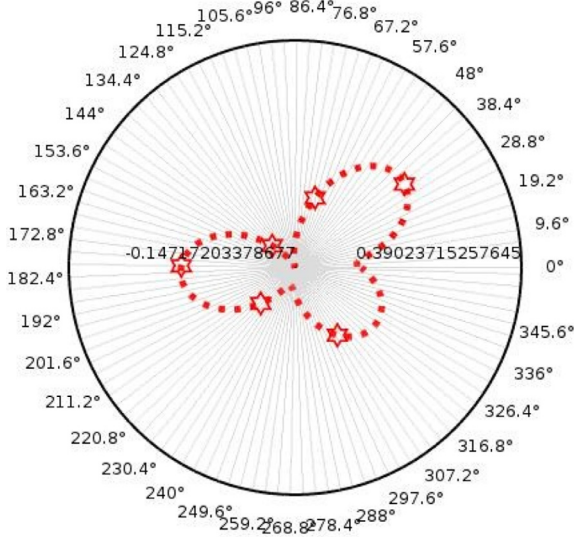


Fig. 3. The electrical energy of $\phi(e_{r(2)})$ in the electric field E .

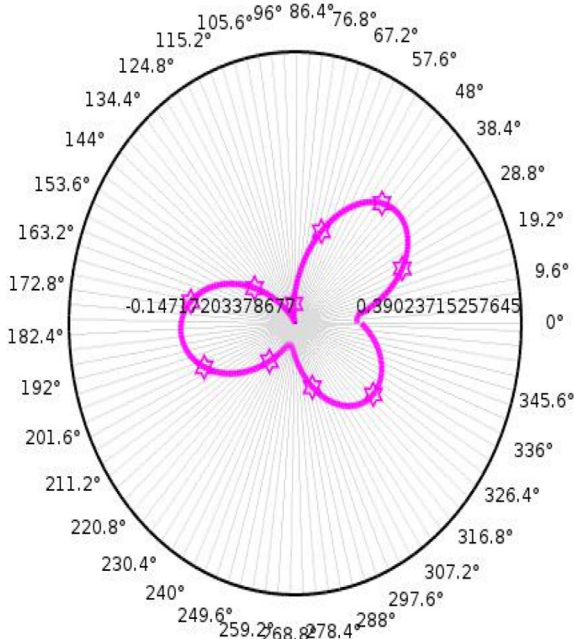


Fig. 4. Potential energy of the energy flux density.

In this section, we construct an orthogonal modern loop in a uniform electric and magnetic field, an electric dipole on the center of a twisting geometric framework and a magnetic dipole on the vector field of a protracted solenoid; in exclusive case, the hidden quasi momentum completely balances the electromagnetic quasi momentum, so the total is zero, logical with the center of energy flux. Ampere's law $\nabla \times B = \mu_0 j$ to recover the modern density in the ordinary expression of hidden quasi momentum, we get

$$p_{hid} = -\epsilon_0 \int (\mathcal{E} \times B) d\tau. \quad (7.1)$$

Theorem 7.1. In Heisenberg space, hidden momentum is given by

$$p_{hid} = -\epsilon_0 \int (((\chi_1(\sigma \chi_1 + \chi_2) + \chi_2(\sigma \chi_2 + \chi_1)) \sin \varphi \sin [\chi_1 s + \chi_2]$$

$$+ \varpi(\sigma \chi_2 + \chi_1) \cos [\chi_1 s + \chi_2] - \varpi(\sigma \chi_1 + \chi_2) \cos \varphi \sin [\chi_1 s + \chi_2]) \mathbf{p}_1 + ((\chi_1(\sigma \chi_1 + \chi_2) + \chi_2(\sigma \chi_2 + \chi_1)) \sin \varphi \cos [\chi_1 s + \chi_2] - \varpi(\sigma \chi_2 + \chi_1) \sin [\chi_1 s + \chi_2] - \varpi(\sigma \chi_1 + \chi_2) \cos \varphi \cos [\chi_1 s + \chi_2]) \mathbf{p}_2 + ((\chi_1(\sigma \chi_1 + \chi_2) + \chi_2(\sigma \chi_2 + \chi_1)) \cos \varphi + \varpi(\sigma \chi_1 + \chi_2) \sin \varphi) \mathbf{p}_3 d\tau. \quad (7.2)$$

Proof. From above integral, the magnetic or electric field is turned off, the field quasi momentum expires, the hidden quasi momentum is consumed, and a few elements in the system may hold an influence. Also, there is no distinct sense why this impulse should equivalent the momentum originally stored in the fields is zero. Thus, hidden quasi momentum is fixed iff

$$((\chi_1(\sigma \chi_1 + \chi_2) + \chi_2(\sigma \chi_2 + \chi_1)) \sin \varphi \sin [\chi_1 s + \chi_2] + \varpi(\sigma \chi_2 + \chi_1) \cos [\chi_1 s + \chi_2] - \varpi(\sigma \chi_1 + \chi_2) \cos \varphi \sin [\chi_1 s + \chi_2]) = 0, ((\chi_1(\sigma \chi_1 + \chi_2) + \chi_2(\sigma \chi_2 + \chi_1)) \sin \varphi \cos [\chi_1 s + \chi_2] - \varpi(\sigma \chi_2 + \chi_1) \sin [\chi_1 s + \chi_2] - \varpi(\sigma \chi_1 + \chi_2) \cos \varphi \cos [\chi_1 s + \chi_2]) = 0, ((\chi_1(\sigma \chi_1 + \chi_2) + \chi_2(\sigma \chi_2 + \chi_1)) \cos \varphi + \varpi(\sigma \chi_1 + \chi_2) \sin \varphi) = 0. \quad (7.3)$$

This completes the proof. \square

8. Conclusion

The comprehensive applications for optical electromagnetic quasi momentum constructions in distinctive areas of optics and technology. By utilize magnetic quasi momentum, principal flexible material researchers have determined modern types of machinery in theoretical physics, energy systems, optics, and constitutional materials.

Magnetic flux executes an essential part in geometric design and style, applied physics, and structural motion.

The main novelty of this paper is to construct a new relationship between Fermi Walker parallel transportation and uniformly quasi accelerated motion in the Heisenberg space.

In our work, we show that energy and magnetic flux density can be stated by Frenet fields in Heisenberg space. Moreover, we obtain potential energy in the cold plasma with respect to its electric field and energy flux density in the radial direction. In our possible future work beneath this kind of idea, we offer to review the magnetic torque of velocity magnetic particles in Heisenberg space.

Declaration of Competing Interest

There are no political, personal, religious, ideological, academic, and intellectual competing interests. The authors declare that they have no competing interests.

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