scientific reports

OPEN

Optical hausdorff quantum energy of spherical magnetic particles

Talat Körpinar[1], Zeliha Körpinar[2], Hatice özdemir[1], ALi Akgül[3,4,5,7]✉ & Murad Khan Hassani[6]✉

In this article, a new approach for spherical magnetic curves under the spherical system in spherical space is given. Firstly, the Hausdorff derivative of the Lorentz spherical magnetic fields $\phi(\beta), \phi(t)$, $\phi(n)$ of spherical magnetic curves is constructed. On the other hand, the Lorentz spherical magnetic fields, by considering the Hausdorff derivative definition, are presented. Eventually, the Hausdorff energies of these spherical Lorentz fields according to the spherical system in $\mathbb{S}^2$ spherical space are computed.

Keywords Spherical magnetic curve, Lorentz magnetic field, Hausdorff derivative, Hausdorff energy

One of the most attractive ways to explain and describe the ordinary space we live in is to study the motions of particles and objects in space. Many of the common methods encountered in geometrical and mathematical research on this motion originate from the field of differential geometry. We can give geometrical systems such as the spherical frame, S-F frame, parallel frame as significant differential geometry methods used to investigate the geometric features of a charged particle. These systems help characterize motion of particles, with the help of the typical properties of the particle, such as torsion and curvature[1–10]. Especially in recent years, many studies based on a spherical frame have been carried out in fields such as applied mathematics and physics. For example, in[11] the geometrical phases worked by Heisenberg anti-ferromagnetic models of potential flows have been obtained. Moreover, in[12], authors have presented some different conditions of spherical electrical flux in an anti-ferromagnetic model and investigated the magnetic $\mathbb{S}_\alpha-$phase of spherical $\mathbb{S}_\alpha-$fluids is accomplished in $\mathbb{S}_\alpha-$fluid particles by geometric results with anti-ferromagnetic concept. On the other hand, in[13], a different approach to spherical $\alpha-$magnetic fibers with some geometrical conditions in Heisenberg space is introduced and some relationships between optical spherical density and spherical optical energy for the Lorentz magnetic field with some approaches to obtain electroosmotic Lorentz force phase is given.

Magnetic curves in ordinary space are trajectories characterized by the charged particle acting according to the influence of the magnetic vector field. When these magnetic curves enter a magnetic vector field, they are exposed to a force called a Lorentz magnetic field. The Lorentz formula is defined by

$$\boldsymbol{V} \times \boldsymbol{X} = \phi(\boldsymbol{X}).$$

On the other hand, the magnetic trajectories of $\boldsymbol{V}$ vector field are presented by

$$\boldsymbol{V} \times \boldsymbol{t} = \phi(\boldsymbol{t}) = D_{\boldsymbol{t}}\boldsymbol{t}.$$

Moreover, in[14,15], the gravitational and frictional magnetic particles in Riemann manifold have been characterized. The magnetic particles have applications in many fields, like optical geometry, physics, physics, differential geometry, biology, etc.[16–18]. Also, if the manifold is 3-dimensional, we can express the above equation thanks to vector multiplication. Recently, magnetic curve characterizations in different spaces have been investigated thanks to many different manifolds using this method[19–31].

The Hausdorff derivative as an alternative approach to fractional calculus, which is equivalent to the defined fractal derivative, moreover, contributes to geometric studies[32–34]. Recently, the Hausdorff derivative has

[1]Department of Mathematics, Muş Alparslan University, 49250 Muş, Turkey. [2]Department of Administration, Muş Alparslan University, 49250 Muş, Turkey. [3]Department of Electronics and Communication Engineering, Saveetha School of Engineering, SIMATS, Chennai, Tamilnadu 602105, India. [4]Department of Computer Engineering, Biruni University, Topkapı, Istanbul 34010, Turkey. [5]Department of Mathematics, Mathematics Research Center, Near East University, Near East Boulevard, Nicosia, Mersin PC: 99138, Turkey. [6]Department of Mathematics, Ghazni University, Ghazni, Afghanistan. [7]Art and Science Faculty, Department of Mathematics, Siirt University, 56100 Siirt, Türkiye. ✉email: aliakgul00727@gmail.com; mhassani@gu.edu.af

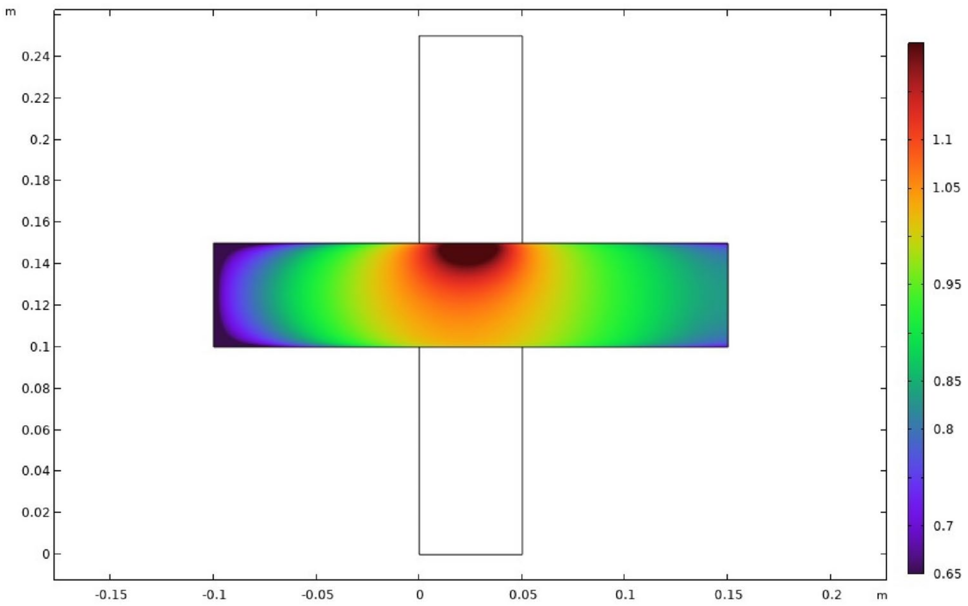

Fig. 1. Hausdorff energy density peaks of $\boldsymbol{n}-$ magnetic $\phi\left(\boldsymbol{\beta}\right)$ for $\sigma=0.5$.

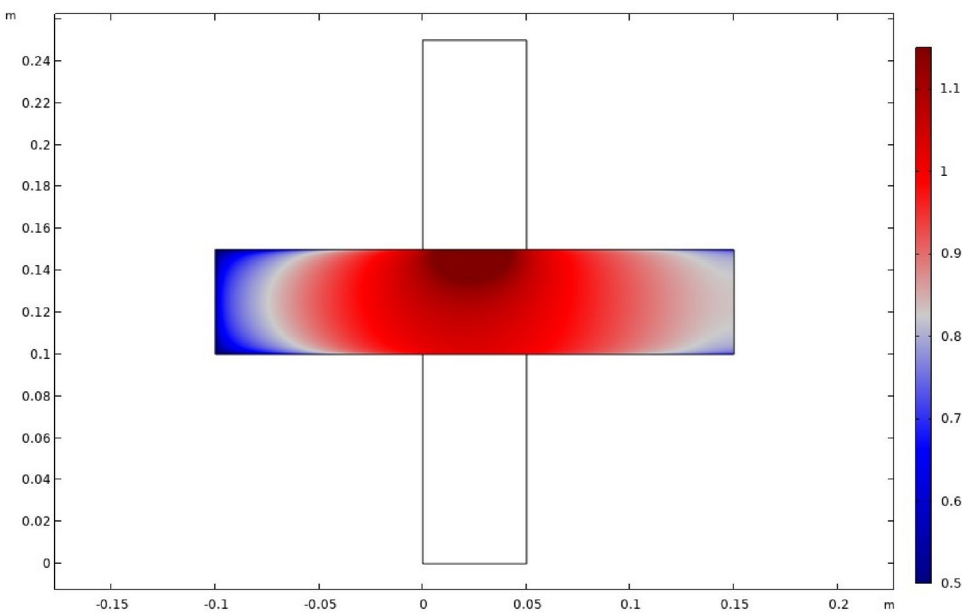

Fig. 2. Hausdorff energy density peaks of $\boldsymbol{n}-$ magnetic $\phi\left(\boldsymbol{t}\right)$ for $\sigma=0.6$.

attracted great attention from authors. Moreover, this derivative is used in different complicated topics in science and engineering[35–41].

This article is organized as follows: First, an approach for spherical magnetic curves associated with spherical systems in space is given. The Hausdorff derivative of the Lorentz spherical magnetic fields $\phi(\boldsymbol{\beta})$, $\phi\left(\boldsymbol{t}\right)$, $\phi\left(\boldsymbol{n}\right)$ of spherical magnetic curves is constructed. On the other hand, the Lorentz spherical magnetic fields, by considering the Hausdorff derivative definition, are presented. Then, the Hausdorff energies of these spherical Lorentz fields according to the spherical system in $\mathbb{S}^2$ spherical space are constructed in Figs. 1, 2, 3, 4.

Spherical magnetic curves

In this section, we presented the basic geometrical construction of magnetic curves and described the Hausdorff derivatives of Lorentz magnetic fields of spherical magnetic curves under the spherical system in $\mathbb{S}^2$ space.

$*$ *The definition of the Hausdorff derivative is characterized by* [42]

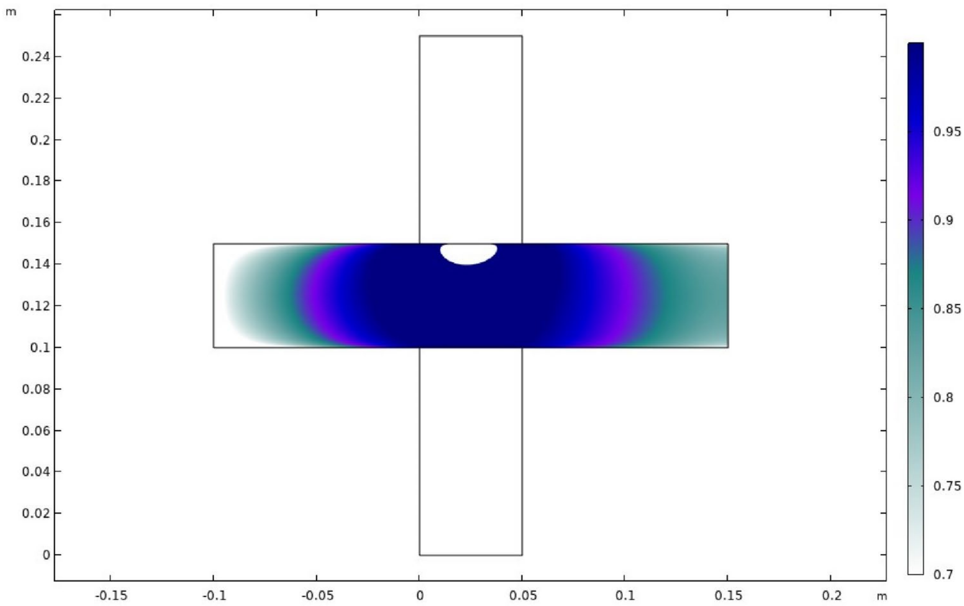

Fig. 3. Hausdorff energy density peaks of $\boldsymbol{n}-$ magnetic $\phi(\boldsymbol{n})$ for $\sigma=0.7$.

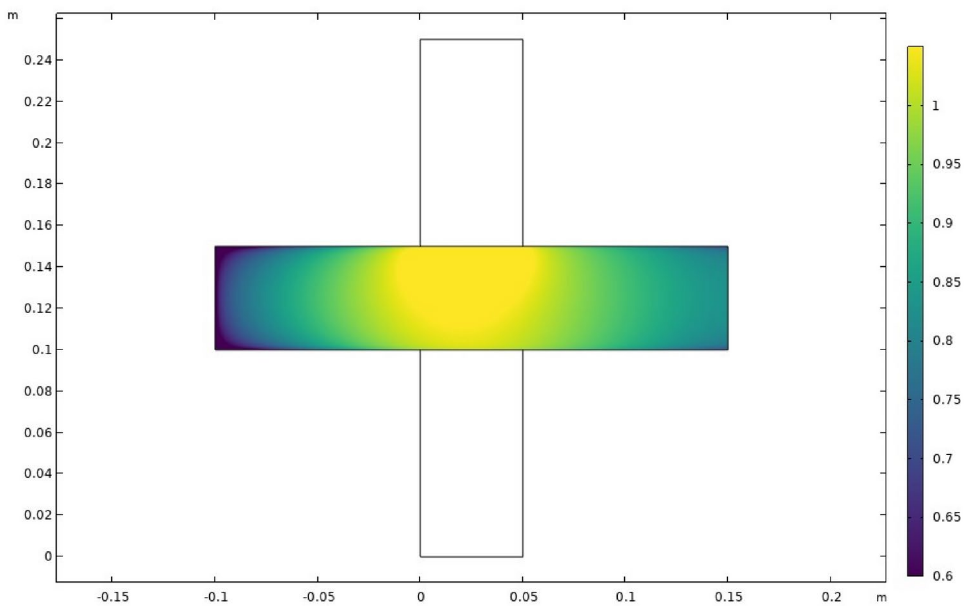

Fig. 4. Hausdorff energy density peaks of $\boldsymbol{n}-$ magnetic $\boldsymbol{V}$ for $\sigma=0.8$.

$$({}_{\mathcal{H}}\mathcal{D}^{\star}f)(t^{\sigma}) = \lim_{t^{'}\to t}\frac{f(t)-f(t^{'})}{t^{\sigma}-t^{'\sigma}} = \frac{t^{1-\sigma}}{\sigma}\,{}_{\mathcal{H}}\mathcal{D}^{\star}f(t).$$

The Hausdorff derivatives for Lorentz forces of the magnetic curve according to the spherical system are given by

$$\begin{aligned} {}_{\mathcal{H}}\mathcal{D}^{\star}\boldsymbol{\beta} &= \frac{1}{\sigma}v^{1-\sigma}\boldsymbol{t}, \\ {}_{\mathcal{H}}\mathcal{D}^{\star}\boldsymbol{t} &= -v^{1-\sigma}\frac{1}{\sigma}\boldsymbol{\beta} + v^{1-\sigma}\mu\frac{1}{\sigma}\boldsymbol{n}, \\ {}_{\mathcal{H}}\mathcal{D}^{\star}\boldsymbol{n} &= -\mu\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{t}, \end{aligned} \tag{1}$$

and where is $\mu=\det\left(\boldsymbol{\beta},\boldsymbol{t},{}_{\mathcal{H}}\mathcal{D}^{\star}\boldsymbol{t}\right)$.

The vector products of magnetic fields are presented as follows

$$\boldsymbol{t}\times\boldsymbol{n}=\boldsymbol{\beta},\boldsymbol{n}\times\boldsymbol{\beta}=\boldsymbol{t},\boldsymbol{\beta}\times\boldsymbol{t}=\boldsymbol{n}.$$

$*$ *Let $\boldsymbol{\beta}$ be a regular curve with a magnetic vector field $\boldsymbol{V}$ in $\mathbb{S}^2$ spherical space. We will call the $\boldsymbol{\beta}$ curve satisfying the following equation, the spherical $\boldsymbol{\beta}-$magnetic curve of the spherical vector field*

$$_{\mathcal{H}}\mathcal{D}^{\bigstar}\boldsymbol{\beta}=\phi\left(\boldsymbol{\beta}\right)=\boldsymbol{V}\times\boldsymbol{\beta}. \tag{2}$$

By using Eqs. (1) and (2), we easily get following electromagnetic fields.

$*$ *The ϕ Lorentz magnetic field for the spherical $\boldsymbol{\beta}$-magnetic curve is given as follows*

$$\begin{aligned}\phi(\boldsymbol{\beta})&=\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{t},\\ \phi(\boldsymbol{t})&=-\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{\beta}+\varkappa_1\boldsymbol{n},\\ \phi(\boldsymbol{n})&=-\varkappa_1\boldsymbol{t},\end{aligned} \tag{3}$$

where $\varkappa_1=\langle\phi\left(\boldsymbol{t}\right),\boldsymbol{n}\rangle$ represents a sufficiently smooth potential. Also, magnetic field $\boldsymbol{V}$ is described by

$$\boldsymbol{V}=\varkappa_1\boldsymbol{\beta}+\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{n}.$$

$*$ *Let $\boldsymbol{\beta}$ be a regular curve with a magnetic vector field $\boldsymbol{V}$ in $\mathbb{S}^2$ spherical space. We will call the $\boldsymbol{\beta}$ curve satisfying the following equation, the spherical $\boldsymbol{t}-$magnetic curve of the spherical vector field*

$$_{\mathcal{H}}\mathcal{D}^{\bigstar}\boldsymbol{t}=\phi\left(\boldsymbol{t}\right)=\boldsymbol{V}\times\boldsymbol{t}. \tag{4}$$

By using Eqs. (1) and (4), we easily get following electromagnetic fields.

$*$ *The ϕ Lorentz magnetic field of the spherical $\boldsymbol{t}$-magnetic curve is obtained as follows*

$$\begin{aligned}\phi(\boldsymbol{\beta})&=\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{t}+\varkappa_2\boldsymbol{n},\\ \phi(\boldsymbol{t})&=-\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{\beta}+\mu\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{n},\\ \phi(\boldsymbol{n})&=-\varkappa_2\boldsymbol{\beta}-\mu\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{t},\end{aligned} \tag{5}$$

where $\varkappa_2=\langle\phi\left(\boldsymbol{\beta}\right),\boldsymbol{n}\rangle$ represents a sufficiently smooth potential. Also, the magnetic vector field $\boldsymbol{V}$ is presented by

$$\boldsymbol{V}=-\varkappa_2\boldsymbol{t}+\mu\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{\beta}+\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{n}.$$

$*$ *Let $\boldsymbol{\beta}$ be a regular curve with a magnetic vector field $\boldsymbol{V}$ in $\mathbb{S}^2$ spherical space. We will call the $\boldsymbol{\beta}$ curve satisfying the following equation, the spherical $\boldsymbol{n}-$magnetic curve of the spherical vector field*

$$_{\mathcal{H}}\mathcal{D}^{\bigstar}\boldsymbol{n}=\phi\left(\boldsymbol{n}\right)=\boldsymbol{V}\times\boldsymbol{n}. \tag{6}$$

By using Eqs. (1) and (6), we easily get following electromagnetic fields.

$*$ *The ϕ Lorentz magnetic field of the spherical $\boldsymbol{n}$-magnetic curve is presented as follows*

$$\begin{aligned}\phi(\boldsymbol{\beta})&=\varkappa_3\boldsymbol{t},\\ \phi(\boldsymbol{t})&=-\varkappa_3\boldsymbol{\beta}+\frac{1}{\sigma}v^{1-\sigma}\mu\boldsymbol{n},\\ \phi(\boldsymbol{n})&=-\mu\frac{1}{\sigma}v^{1-\sigma}\boldsymbol{t},\end{aligned} \tag{7}$$

where $\varkappa_3=\langle\phi\left(\boldsymbol{\beta}\right),\boldsymbol{t}\rangle$ represents a sufficiently smooth potential. Moreover, the magnetic vector field $\boldsymbol{V}$ is described by

$$\boldsymbol{V}=\frac{1}{\sigma}v^{1-\sigma}\mu\boldsymbol{\beta}+\varkappa_3\boldsymbol{n}.$$

Hausdorff energies of Lorentz forces of spherical magnetic curves

In this part, we investigated Hausdorff energies of normalized and recursional magnetic fields of timelike magnetic curves associated with the Bishop system in space.

$\beta-$magnetic curves

★ *The Hausdorff derivatives of ϕ Lorentz force for a spherical $\beta-$ magnetic curve associated with a spherical system are obtained by*

$$
\begin{aligned}
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi(\boldsymbol{\beta}) &= \frac{1}{\sigma^2}\mu v^{2-2\sigma}\boldsymbol{n} \\
&\quad + (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\boldsymbol{t} - v^{-2\sigma+2}\frac{1}{\sigma^2}\boldsymbol{\beta}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi(\boldsymbol{t}) &= (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\boldsymbol{\beta} \\
&\quad + v^{1-\sigma}\frac{1}{\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_1)\boldsymbol{n} - \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1\mu + \frac{1}{\sigma^2}v^{2-2\sigma}\right)\boldsymbol{t}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi(\boldsymbol{n}) &= -\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_1)\boldsymbol{t} \\
&\quad - \frac{1}{\sigma}\varkappa_1 v^{1-\sigma}\mu\boldsymbol{n} + \varkappa_1 v^{1-\sigma}\frac{1}{\sigma}\boldsymbol{\beta}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\boldsymbol{V} &= v^{1-\sigma}\frac{1}{\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_1)\boldsymbol{\beta} \\
&\quad + \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1 - \frac{1}{\sigma}v^{1-\sigma}\mu\right)\boldsymbol{t}.
\end{aligned}
$$

★ *The Hausdorff energies of ϕ Lorentz force of a $\beta-$magnetic curve under the spherical system are described as follows:*

$$
\begin{aligned}
\varepsilon nergy(\phi(\boldsymbol{\beta})) &= \frac{1}{2}\int_{\sigma}\left(1 + \left(v^{-2\sigma+2}\frac{1}{\sigma^2}\mu\right)^2\right. \\
&\quad \left. + \left((1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + \left(v^{-2\sigma+2}\frac{1}{\sigma^2}\right)^2\right)dv, \\
\varepsilon nergy(\phi(\boldsymbol{t})) &= \frac{1}{2}\int_{\sigma}\left(1 + \frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_1)^2 v^{2-2\sigma}\right. \\
&\quad \left. + \left((1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1\mu + \frac{1}{\sigma^2}v^{2-2\sigma}\right)^2\right)dv, \\
\varepsilon nergy(\phi(\boldsymbol{n})) &= \frac{1}{2}\int_{\sigma}\left(1 + v^{2-2\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_1)^2\frac{1}{\sigma^2}\right. \\
&\quad \left. + v^{2-2\sigma}\varkappa_1^2\frac{1}{\sigma^2} + \varkappa_1^2\frac{1}{\sigma^2}\mu^2 v^{2-2\sigma}\right)dv, \\
\varepsilon nergy(\boldsymbol{V}) &= \frac{1}{2}\int_{\sigma}\left(1 + \frac{1}{\sigma^2}v^{2-2\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_1)^2\right. \\
&\quad \left. + \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1 - \frac{1}{\sigma}v^{1-\sigma}\mu\right)^2\right)dv.
\end{aligned}
$$

$\boldsymbol{t}-$magnetic curves

★ *The Hausdorff derivatives of ϕ Lorentz force for a spherical $\boldsymbol{t}-$magnetic curve associated with a spherical system are given by*

$$
\begin{aligned}
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi\left(\boldsymbol{\beta}\right) &= -v^{-2\sigma+2}\frac{1}{\sigma^2}\boldsymbol{\beta} + \Big(-\frac{1}{\sigma}\mu v^{1-\sigma}\varkappa_2 \\
&\quad + (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)\boldsymbol{t} + \Big(v^{2-2\sigma}\mu\frac{1}{\sigma^2} + \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_2)\Big)\boldsymbol{n}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi\left(\boldsymbol{t}\right) &= (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\boldsymbol{\beta} - \Big(\frac{1}{\sigma^2}\mu^2 v^{-2\sigma+2} \\
&\quad + v^{2-2\sigma}\frac{1}{\sigma^2}\Big)\boldsymbol{t} + \Big(v^{2-2\sigma}\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu) + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)\boldsymbol{n}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi\left(\boldsymbol{n}\right) &= \Big(\mu\frac{1}{\sigma^2}v^{2-2\sigma} - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_2)\Big)\boldsymbol{\beta} \\
&\quad - \Big(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu)v^{2-2\sigma} + \frac{1}{\sigma}\varkappa_2 v^{1-\sigma} + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)\boldsymbol{t} - \frac{1}{\sigma^2}\mu^2 v^{2-2\sigma}\boldsymbol{n}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\boldsymbol{V} &= \Big(\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu) + \frac{1}{\sigma}v^{1-\sigma}\varkappa_2\Big)\boldsymbol{\beta} \\
&\quad - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_2)\boldsymbol{t} - \frac{1}{\sigma}v^{1-\sigma}\varkappa_2\mu\boldsymbol{n}.
\end{aligned}
$$

★ *The Hausdorff energies of* ϕ *Lorentz force of a* $\boldsymbol{t}-$*magnetic curve under the spherical system are obtained as follows:*

$$
\begin{aligned}
\varepsilon nergy\left(\phi\left(\boldsymbol{\beta}\right)\right) &= \frac{1}{2}\int_{\sigma}\Big(1 + (\frac{1}{\sigma^2}v^{2-2\sigma})^2 + \Big(-\frac{1}{\sigma}\mu v^{1-\sigma}\varkappa_2 \\
&\quad + (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)^2 + \Big(\frac{1}{\sigma^2}\mu v^{2-2\sigma} + \frac{1}{\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_2)v^{1-\sigma}\Big)^2\Big)\,dv, \\
\varepsilon nergy\left(\phi\left(\boldsymbol{t}\right)\right) &= \frac{1}{2}\int_{\sigma}\Big(1 + \Big(\frac{1}{\sigma^2}\mu^2 v^{-2\sigma+2} + v^{2-2\sigma}\frac{1}{\sigma^2}\Big)^2 \\
&\quad \Big((1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)^2 + \Big(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu)v^{2-2\sigma}\Big)^2\Big)\,dv, \\
\varepsilon nergy\left(\phi\left(\boldsymbol{n}\right)\right) &= \frac{1}{2}\int_{\sigma}\Big(1 + (\mu\frac{1}{\sigma^2}v^{2-2\sigma} - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_2))^2 \\
&\quad + \Big(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu)v^{2-2\sigma} + \frac{1}{\sigma}\varkappa_2 v^{1-\sigma} + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)^2 + (v^{2-2\sigma}\frac{1}{\sigma^2}\mu^2)^2\Big)\,dv, \\
\varepsilon nergy\left(\boldsymbol{V}\right) &= \frac{1}{2}\int_{\sigma}\Big(1 + \Big(\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu) + \frac{1}{\sigma}v^{1-\sigma}\varkappa_2\Big)^2 \\
&\quad + \frac{1}{\sigma^2}v^{2-2\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_2)^2 + \mu^2\frac{1}{\sigma^2}v^{2-2\sigma}\varkappa_2^2\Big)\,dv.
\end{aligned}
$$

$\boldsymbol{n}-$magnetic curves

★ *The Hausdorff derivatives of* ϕ *Lorentz force for a spherical* $\boldsymbol{n}-$*magnetic curve associated with a spherical system are obtained by*

$$
\begin{aligned}
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi\left(\boldsymbol{\beta}\right) &= \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_3)\boldsymbol{t} \\
&\quad + \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\mu\boldsymbol{n} - \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\boldsymbol{\beta}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi\left(\boldsymbol{t}\right) &= \Big(\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu) + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)\boldsymbol{n} \\
&\quad - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_3)\boldsymbol{\beta} - \Big(\frac{1}{\sigma^2}v^{2-2\sigma}\mu^2 + \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\Big)\boldsymbol{t}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\phi\left(\boldsymbol{n}\right) &= v^{2-2\sigma}\mu\frac{1}{\sigma^2}\boldsymbol{\beta} - \Big(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu)v^{2-2\sigma} \\
&\quad + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\Big)\boldsymbol{t} - \frac{1}{\sigma^2}\mu^2 v^{2-2\sigma}\boldsymbol{n}, \\
{}_{\mathcal{H}}\mathcal{D}^{\bigstar}\boldsymbol{V} &= \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\mu)\boldsymbol{\beta} + \Big(\frac{1}{\sigma^2}v^{2-2\sigma}\mu \\
&\quad - \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\mu\Big)\boldsymbol{t} + \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\bigstar}\varkappa_3)\boldsymbol{n}.
\end{aligned}
$$

★ *The Hausdorff energies of* ϕ *Lorentz force of a* $\boldsymbol{n}-$*magnetic curve under the spherical system are constructed as follows:*

$$\varepsilon nergy\left(\phi\left(\boldsymbol{\beta}\right)\right)=\frac{1}{2}\int_{\sigma}\left(1+\left(\frac{1}{\sigma}v^{1-\sigma}\left({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_{3}\right)\right)^{2}+\frac{1}{\sigma^{2}}v^{2-2\sigma}\varkappa_{3}^{2}\mu^{2}+\frac{1}{\sigma^{2}}v^{2-2\sigma}\varkappa_{3}^{2}\right)dv,$$

$$\varepsilon nergy\left(\phi\left(\boldsymbol{t}\right)\right)=\frac{1}{2}\int_{\sigma}\left(1+\left(\frac{1}{\sigma}v^{1-\sigma}\left({}_{\mathcal{H}}\mathcal{D}^{\star}\mu\right)+\mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^{2}+\frac{1}{\sigma^{2}}v^{2-2\sigma}\left({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_{3}\right)^{2}+\left(\frac{1}{\sigma^{2}}v^{2-2\sigma}\mu^{2}+\frac{1}{\sigma}v^{1-\sigma}\varkappa_{3}\right)^{2}\right)dv,$$

$$\varepsilon nergy\left(\phi\left(\boldsymbol{n}\right)\right)=\frac{1}{2}\int_{\sigma}\left(1+\left(\frac{1}{\sigma^{2}}\mu v^{-2\sigma+2}\right)^{2}+\left(\frac{1}{\sigma^{2}}\left({}_{\mathcal{H}}\mathcal{D}^{\star}\mu\right)v^{2-2\sigma}+\mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^{2}+\left(\frac{1}{\sigma^{2}}\mu^{2}v^{-2\sigma+2}\right)^{2}\right)dv,$$

$$\varepsilon nergy\left(\boldsymbol{V}\right)=\frac{1}{2}\int_{\sigma}\left(1+\frac{1}{\sigma^{2}}v^{2-2\sigma}\left({}_{\mathcal{H}}\mathcal{D}^{\star}\mu\right)^{2}+\left(\frac{1}{\sigma}v^{1-\sigma}\mu-\frac{1}{\sigma}v^{1-\sigma}\varkappa_{3}\mu\right)^{2}+\frac{1}{\sigma^{2}}v^{2-2\sigma}\left({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_{3}\right)^{2}\right)dv.$$

Applications for energy density peaks
The graphical and energy analysis of $\boldsymbol{n}-$magnetic curve is presented in Figs. 1, 2, 3 and 4, providing insights into the performance of Hausdorff energy density peaks in conformable model.

Conclusions
In this article, an approach for spherical magnetic curves under the spherical system in space are given. Firstly, the Hausdorff derivative of the Lorentz spherical magnetic fields $\phi(\boldsymbol{\beta})$, $\phi\left(\boldsymbol{t}\right)$, $\phi\left(\boldsymbol{n}\right)$ of spherical magnetic curves is constructed. On the other hand, the Lorentz spherical magnetic fields by considering Hausdorff derivative definition are presented. Eventually, the Hausdorff energies of these spherical Lorentz magnetic fields $\phi(\boldsymbol{\beta})$, $\phi\left(\boldsymbol{t}\right)$, $\phi\left(\boldsymbol{n}\right)$ associated with the spherical system in $\mathbb{S}^2$ spherical space are computed .

Data availibility
Data will be provided by corresponding author on reasonable request.

Received: 30 July 2024; Accepted: 24 October 2024
Published online: 04 February 2025

References
1. Lamb Jr, G. L. Elements of Soliton Theory. New York, 29. (1980).
2. Ablowitz, M. J. & Clarkson, P. A. Solitons, Nonlinear Evolution Equations and Inverse Scattering (Vol. 149). Cambridge university press. (1991).
3. Matveev, V. B. & Salle, M. A. *Darboux Transformations and Solitons* Vol. 17 (Springer, Berlin, 1991).
4. Wang, M., Zhou, Y. & Li, Z. Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics. *Phys. Lett. A* **216**(1–5), 67–75 (1996).
5. Lakshmanan, M. & Rajaseekar, S. *Nonlinear Dynamics: Integrability, Chaos and Patterns* (Springer Science & Business Media, 2012).
6. Krishnan, E. V. & Biswas, A. Solutions to the Zakharov–Kuznetsov equation with higher order nonlinearity by mapping and ansatz methods. *Phys. Wave Phenomena* **18**(4), 256–261 (2010).
7. Hasimoto, H. A soliton on a vortex filament. *J. Fluid Mech.* **51**(3), 477–485 (1972).
8. Körpinar, T. & Demirkol, R. C. Berry phase of the linearly polarized light wave along an optical fiber and its electromagnetic curves via quasi adapted frame. *Waves Random Complex Media* **32**(3), 1497–1516 (2022).
9. Soliman, M. A., Abdel-All, N. H., Hussien, R. A. & Youssef, T. Evolutions of the ruled surfaces via the evolution of their directrix using quasi frame along a space curve. *J. Appl. Math. Phys.* **6**(08), 1748 (2018).
10. Körpinar, T. Optical directional binormal magnetic flows with geometric phase: Heisenberg ferromagnetic model. *Optik* **219**, 165134 (2020).
11. Körpinar, T. & Sazak, A. Geometric phase for spherical magnetic particles. *Optik* **273**, 170431 (2023).
12. Körpinar, T., Körpinar, Z. & Asil, V. Electric flux fibers with spherical antiferromagnetic approach with electroosmotic velocity. *Optik* **252**, 168108 (2022).
13. Körpinar, Z. & Körpinar, T. Optical spherical electroosmotic phase and optical energy for spherical $\alpha-$magnetic fibers. *Optik* **255**, 168455 (2022).
14. Korpinar, T. & Demirkol, R. C. Frictional magnetic curves in 3D Riemannian manifolds. *Int. J. Geom. Methods Modern Phys.* **15**(02), 1850020 (2018).
15. Korpınar, T. & Demirkol, R. C. Gravitational magnetic curves on 3D Riemannian manifolds. *Int. J. Geometric Methods Modern Phys.* **15**(11), 1850184 (2018).
16. Barros, M., Cabrerizo, J. L., Fernández, M. & Romero, A. Magnetic vortex filament flows. *J. Math. Phys.* **48**(8), 082904 (2007).
17. Cabrerizo, J. L. Magnetic fields in 2D and 3D sphere. *J. Nonlinear Math. Phys.* **20**(3), 440–450 (2013).
18. Bozkurt, Z., Gök, I., Yaylı, Y. & Ekmekci, F. N. A new approach for magnetic curves in 3D Riemannian manifolds. *J. Math. Phys.* **55**(5), 053501 (2014).
19. Cabrerizo, J. L., Fernández, M. & Gómez, J. S. The contact magnetic flow in 3D Sasakian manifolds. *J. Phys. A: Math. Theor.* **42**(19), 195201 (2009).

20. Sunada, T. Magnetic flows on a Riemannian surface. In Proc. of KAIST Math. Workshop (Vol. 8, pp. 93-108). Analysis and Geometry. (1993).
21. Calin, C. & Crasmareanu, T. Magnetic curves in three-dimensional quasi-para-Sasakian geometry. *Mediterr. J. Math.* **13**, 2087–2097 (2016).
22. Druţa-Romaniuc, S. L., Inoguchi, J. I., Munteanu, M. I. & Nistor, A. I. Magnetic curves in cosymplectic manifolds. *Rep. Math. Phys.* **78**(1), 33–48 (2016).
23. Barros, M., Cabrerizo, J. L., Fernández, M. & Romero, A. Magnetic vortex filament flows. J. Math. Phys., **48**(8) (2007).
24. Bozkurt, Z., Gök, I., Yaylı, Y. & Ekmekci, F. N. A new approach for magnetic curves in 3D Riemannian manifolds. J. Math. Phys., **55** (5) (2014).
25. Körpınar, T. & Demirkol, R. C. Gravitational magnetic curves on 3D Riemannian manifolds. *Int. J. Geom. Methods Modern Phys.* **15**(11), 1850184 (2018).
26. Körpinar, T. Optical electromotive force with Heisenberg spherical ferromagnetic spin. *Optik* **245**, 167521 (2021).
27. Körpinar, T. & Körpinar, Z. Spherical electric and magnetic phase with Heisenberg spherical ferromagnetic spin by some fractional solutions. *Optik* **242**, 167164 (2021).
28. Altın, A. The energy of a domain on the surface. *Ukr. Math. J.* **67**(4), 641–647 (2015).
29. Körpinar, T., Demirkol, R. C., Körpinar, Z. & Asil, V. Fractional solutions for the inextensible Heisenberg antiferromagnetic flow and solitonic magnetic flux surfaces in the binormal direction. *Revista Mexicana de física* **67**(3), 452–464 (2021).
30. Haus, H. A. & Melcher, J. R. *Electromagnetic Fields and Energy* Vol. 107 (Prentice Hall, Englewood Cliffs, 1989).
31. Synge, J. L. Relativity: The General Theory. (1960).
32. Chen, W. Fractal analysis of Hausdorff calculus and fractional calculus models. *Comput.-Aided Eng.* **26**(3), 2–5 (2017).
33. Weberszpil, J., Lazo, M. J. & Helayël-Neto, J. A. On a connection between a class of q-deformed algebras and the Hausdorff derivative in a medium with fractal metric. *Physica A* **436**, 399–404 (2015).
34. Li, J. & Ostoja-Starzewski, M. Fractal solids, product measures and fractional wave equations. *Proc. Royal Soc. A: Math., Phys. Eng. Sci.* **465**(2108), 2521–2536 (2009).
35. Li, J. & Ostoja-Starzewski, M. Comment on "hydrodynamics of fractal continuum flow" and "map of fluid flow in fractal porous medium into fractal continuum flow". *Phys. Rev. E* **88**(5), 057001 (2013).
36. Sun, H., Meerschaert, M. M., Zhang, Y., Zhu, J. & Chen, W. A fractal Richards' equation to capture the non-Boltzmann scaling of water transport in unsaturated media. *Adv. Water Resour.* **52**, 292–295 (2013).
37. Chen, W., Sun, H., Zhang, X. & Korosak, D. Anomalous diffusion modeling by fractal and fractional derivatives. *Comput. Math. Appl.* **59**(5), 1754–1758 (2010).
38. Cai, W., Chen, W. & Xu, W. Characterizing the creep of viscoelastic materials by fractal derivative models. *Int. J. Non-Linear Mech.* **87**, 58–63 (2016).
39. Lin, G. An effective phase shift diffusion equation method for analysis of PFG normal and fractional diffusions. *J. Magn. Reson.* **259**, 232–240 (2015).
40. Lin, G. Instantaneous signal attenuation method for analysis of PFG fractional diffusions. *J. Magn. Reson.* **269**, 36–49 (2016).
41. Reyes-Marambio, J. et al. A fractal time thermal model for predicting the surface temperature of air-cooled cylindrical Li-ion cells based on experimental measurements. *J. Power Sources* **306**, 636–645 (2016).
42. Balankin, A. S. & Mena, B. Vector differential operators in a fractional dimensional space, on fractals, and in fractal continua. *Chaos, Solitons Fract.* **168**, 113203 (2023).

Author contributions

All authors reviewed the manuscript.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

Correspondence and requests for materials should be addressed to A.A. or M.K.H.

Reprints and permissions information is available at www.nature.com/reprints.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.