



OPEN Optical hausdorff quantum energy of spherical magnetic particles

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In this article, a new approach for spherical magnetic curves under the spherical system in spherical space is given. Firstly, the Hausdorff derivative of the Lorentz spherical magnetic fields $\phi(\beta)$, $\phi(t)$, $\phi(n)$ of spherical magnetic curves is constructed. On the other hand, the Lorentz spherical magnetic fields, by considering the Hausdorff derivative definition, are presented. Eventually, the Hausdorff energies of these spherical Lorentz fields according to the spherical system in S^2 spherical space are computed.

Keywords Spherical magnetic curve, Lorentz magnetic field, Hausdorff derivative, Hausdorff energy

One of the most attractive ways to explain and describe the ordinary space we live in is to study the motions of particles and objects in space. Many of the common methods encountered in geometrical and mathematical research on this motion originate from the field of differential geometry. We can give geometrical systems such as the spherical frame, S-F frame, parallel frame as significant differential geometry methods used to investigate the geometric features of a charged particle. These systems help characterize motion of particles, with the help of the typical properties of the particle, such as torsion and curvature^{1–10}. Especially in recent years, many studies based on a spherical frame have been carried out in fields such as applied mathematics and physics. For example, in¹¹ the geometrical phases worked by Heisenberg anti-ferromagnetic models of potential flows have been obtained. Moreover, in¹², authors have presented some different conditions of spherical electrical flux in an anti-ferromagnetic model and investigated the magnetic S_α -phase of spherical S_α -fluids is accomplished in S_α -fluid particles by geometric results with anti-ferromagnetic concept. On the other hand, in¹³, a different approach to spherical α -magnetic fibers with some geometrical conditions in Heisenberg space is introduced and some relationships between optical spherical density and spherical optical energy for the Lorentz magnetic field with some approaches to obtain electroosmotic Lorentz force phase is given.

Magnetic curves in ordinary space are trajectories characterized by the charged particle acting according to the influence of the magnetic vector field. When these magnetic curves enter a magnetic vector field, they are exposed to a force called a Lorentz magnetic field. The Lorentz formula is defined by

$$\mathbf{V} \times \mathbf{X} = \phi(\mathbf{X}).$$

On the other hand, the magnetic trajectories of \mathbf{V} vector field are presented by

$$\mathbf{V} \times \mathbf{t} = \phi(\mathbf{t}) = D_t \mathbf{t}.$$

Moreover, in^{14,15}, the gravitational and frictional magnetic particles in Riemann manifold have been characterized. The magnetic particles have applications in many fields, like optical geometry, physics, physics, differential geometry, biology, etc.^{16–18}. Also, if the manifold is 3-dimensional, we can express the above equation thanks to vector multiplication. Recently, magnetic curve characterizations in different spaces have been investigated thanks to many different manifolds using this method^{19–31}.

The Hausdorff derivative as an alternative approach to fractional calculus, which is equivalent to the defined fractal derivative, moreover, contributes to geometric studies^{32–34}. Recently, the Hausdorff derivative has

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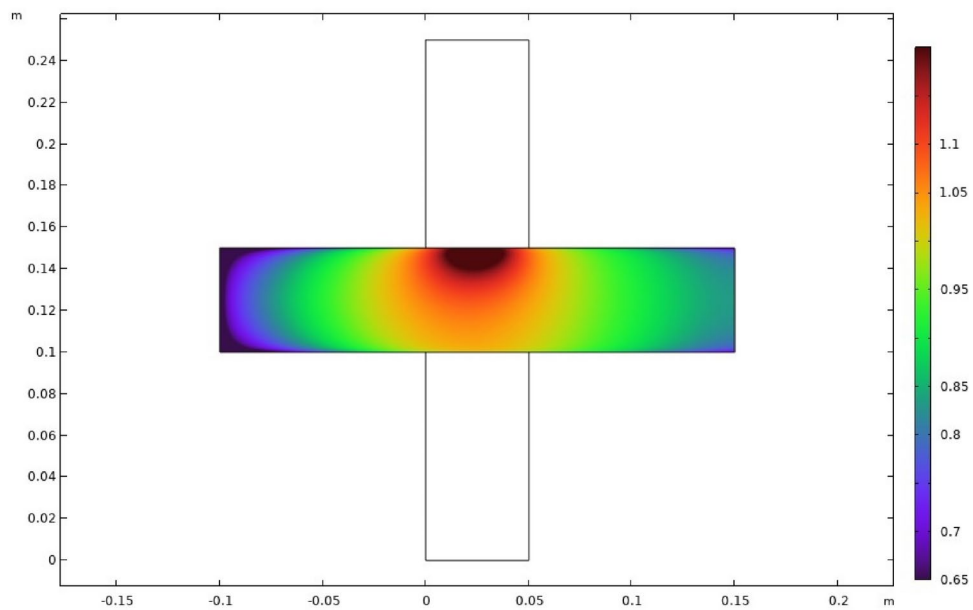


Fig. 1. Hausdorff energy density peaks of n – magnetic $\phi(\beta)$ for $\sigma = 0.5$.

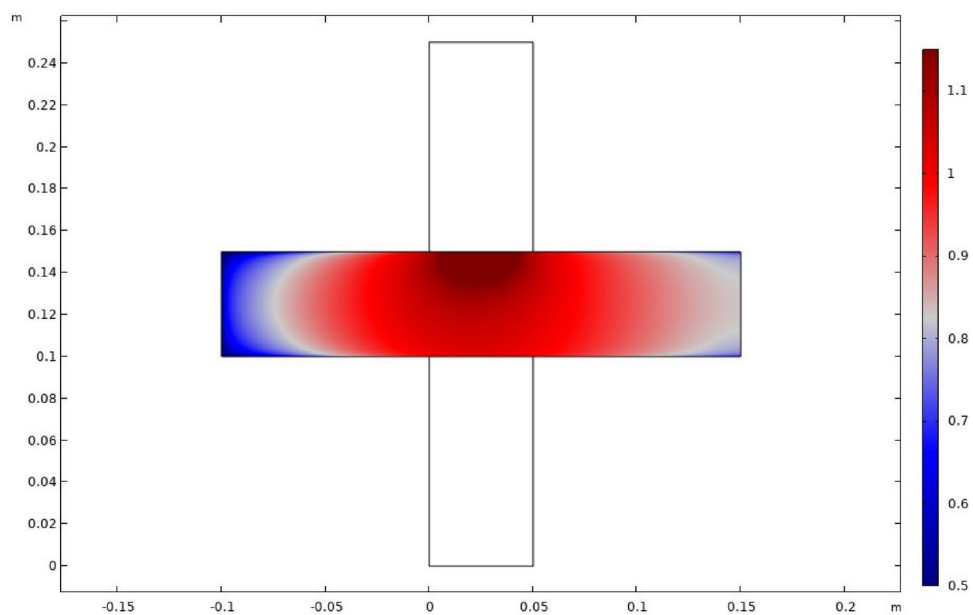


Fig. 2. Hausdorff energy density peaks of n – magnetic $\phi(t)$ for $\sigma = 0.6$.

attracted great attention from authors. Moreover, this derivative is used in different complicated topics in science and engineering^{35–41}.

This article is organized as follows: First, an approach for spherical magnetic curves associated with spherical systems in space is given. The Hausdorff derivative of the Lorentz spherical magnetic fields $\phi(\beta)$, $\phi(t)$, $\phi(n)$ of spherical magnetic curves is constructed. On the other hand, the Lorentz spherical magnetic fields, by considering the Hausdorff derivative definition, are presented. Then, the Hausdorff energies of these spherical Lorentz fields according to the spherical system in S^2 spherical space are constructed in Figs. 1, 2, 3, 4.

Spherical magnetic curves

In this section, we presented the basic geometrical construction of magnetic curves and described the Hausdorff derivatives of Lorentz magnetic fields of spherical magnetic curves under the spherical system in S^2 space.

* The definition of the Hausdorff derivative is characterized by⁴²

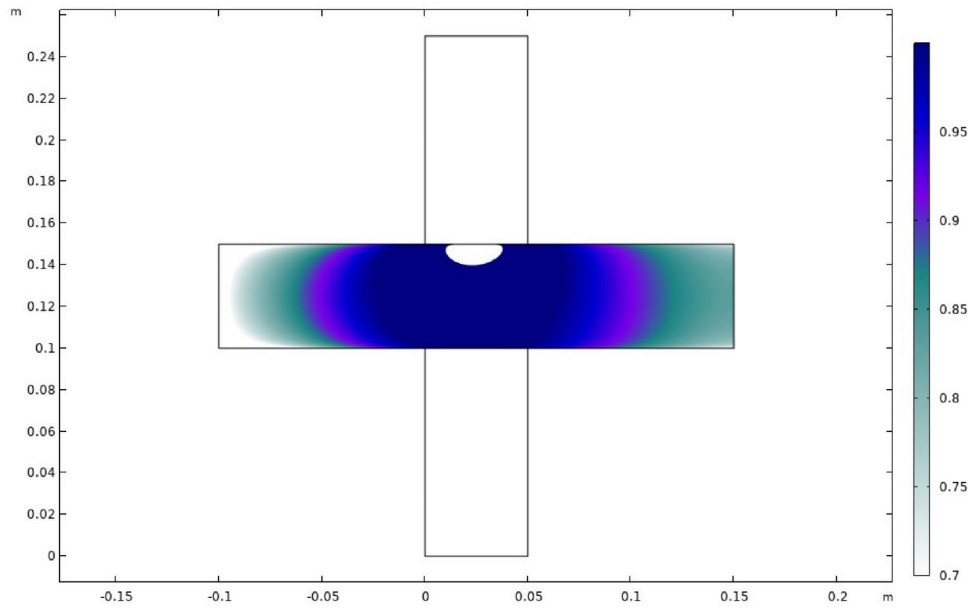


Fig. 3. Hausdorff energy density peaks of $n -$ magnetic $\phi(n)$ for $\sigma = 0.7$.

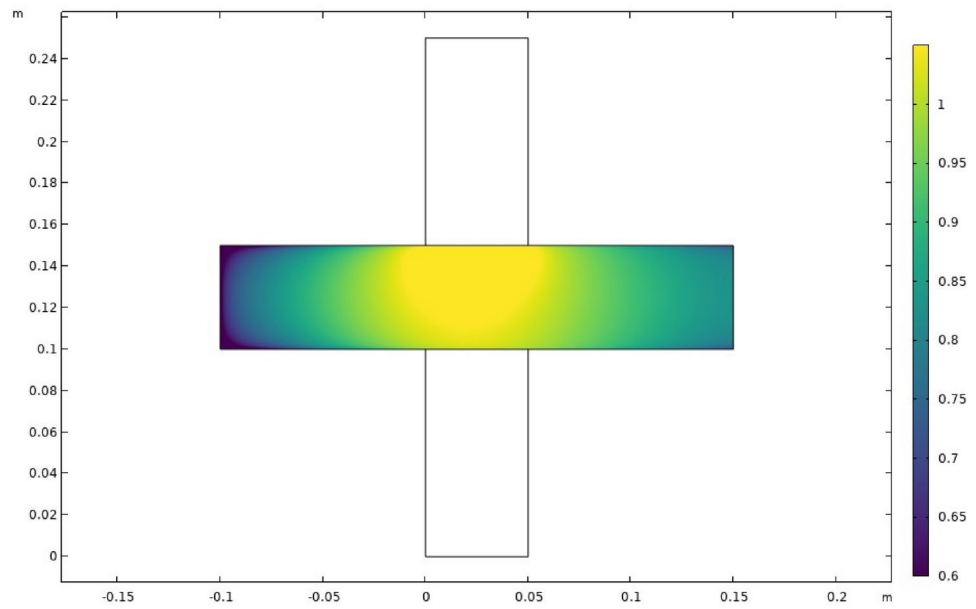


Fig. 4. Hausdorff energy density peaks of $n -$ magnetic V for $\sigma = 0.8$.

$$({}_H D^\star f)(t^\sigma) = \lim_{t' \rightarrow t} \frac{f(t) - f(t')}{t^\sigma - t'^\sigma} = \frac{t^{1-\sigma}}{\sigma} {}_H D^\star f(t).$$

The Hausdorff derivatives for Lorentz forces of the magnetic curve according to the spherical system are given by

$$\begin{aligned} {}_H D^\star \beta &= \frac{1}{\sigma} v^{1-\sigma} t, \\ {}_H D^\star t &= -v^{1-\sigma} \frac{1}{\sigma} \beta + v^{1-\sigma} \mu \frac{1}{\sigma} n, \\ {}_H D^\star n &= -\mu \frac{1}{\sigma} v^{1-\sigma} t, \end{aligned} \tag{1}$$

and where is $\mu = \det(\beta, t, {}_H D^\star t)$.

The vector products of magnetic fields are presented as follows

$$\mathbf{t} \times \mathbf{n} = \boldsymbol{\beta}, \mathbf{n} \times \boldsymbol{\beta} = \mathbf{t}, \boldsymbol{\beta} \times \mathbf{t} = \mathbf{n}.$$

* Let $\boldsymbol{\beta}$ be a regular curve with a magnetic vector field \mathbf{V} in S^2 spherical space. We will call the $\boldsymbol{\beta}$ curve satisfying the following equation, the spherical $\boldsymbol{\beta}$ -magnetic curve of the spherical vector field

$$\mathcal{H}\mathcal{D}^{\star}\boldsymbol{\beta} = \phi(\boldsymbol{\beta}) = \mathbf{V} \times \boldsymbol{\beta}. \quad (2)$$

By using Eqs. (1) and (2), we easily get following electromagnetic fields.

* The ϕ Lorentz magnetic field for the spherical $\boldsymbol{\beta}$ -magnetic curve is given as follows

$$\begin{aligned} \phi(\boldsymbol{\beta}) &= \frac{1}{\sigma} v^{1-\sigma} \mathbf{t}, \\ \phi(\mathbf{t}) &= -\frac{1}{\sigma} v^{1-\sigma} \boldsymbol{\beta} + \varkappa_1 \mathbf{n}, \\ \phi(\mathbf{n}) &= -\varkappa_1 \mathbf{t}, \end{aligned} \quad (3)$$

where $\varkappa_1 = \langle \phi(\mathbf{t}), \mathbf{n} \rangle$ represents a sufficiently smooth potential. Also, magnetic field \mathbf{V} is described by

$$\mathbf{V} = \varkappa_1 \boldsymbol{\beta} + \frac{1}{\sigma} v^{1-\sigma} \mathbf{n}.$$

* Let $\boldsymbol{\beta}$ be a regular curve with a magnetic vector field \mathbf{V} in S^2 spherical space. We will call the $\boldsymbol{\beta}$ curve satisfying the following equation, the spherical \mathbf{t} -magnetic curve of the spherical vector field

$$\mathcal{H}\mathcal{D}^{\star}\mathbf{t} = \phi(\mathbf{t}) = \mathbf{V} \times \mathbf{t}. \quad (4)$$

By using Eqs. (1) and (4), we easily get following electromagnetic fields.

* The ϕ Lorentz magnetic field of the spherical \mathbf{t} -magnetic curve is obtained as follows

$$\begin{aligned} \phi(\boldsymbol{\beta}) &= \frac{1}{\sigma} v^{1-\sigma} \mathbf{t} + \varkappa_2 \mathbf{n}, \\ \phi(\mathbf{t}) &= -\frac{1}{\sigma} v^{1-\sigma} \boldsymbol{\beta} + \mu \frac{1}{\sigma} v^{1-\sigma} \mathbf{n}, \\ \phi(\mathbf{n}) &= -\varkappa_2 \boldsymbol{\beta} - \mu \frac{1}{\sigma} v^{1-\sigma} \mathbf{t}, \end{aligned} \quad (5)$$

where $\varkappa_2 = \langle \phi(\boldsymbol{\beta}), \mathbf{n} \rangle$ represents a sufficiently smooth potential. Also, the magnetic vector field \mathbf{V} is presented by

$$\mathbf{V} = -\varkappa_2 \mathbf{t} + \mu \frac{1}{\sigma} v^{1-\sigma} \boldsymbol{\beta} + \frac{1}{\sigma} v^{1-\sigma} \mathbf{n}.$$

* Let $\boldsymbol{\beta}$ be a regular curve with a magnetic vector field \mathbf{V} in S^2 spherical space. We will call the $\boldsymbol{\beta}$ curve satisfying the following equation, the spherical \mathbf{n} -magnetic curve of the spherical vector field

$$\mathcal{H}\mathcal{D}^{\star}\mathbf{n} = \phi(\mathbf{n}) = \mathbf{V} \times \mathbf{n}. \quad (6)$$

By using Eqs. (1) and (6), we easily get following electromagnetic fields.

* The ϕ Lorentz magnetic field of the spherical \mathbf{n} -magnetic curve is presented as follows

$$\begin{aligned} \phi(\boldsymbol{\beta}) &= \varkappa_3 \mathbf{t}, \\ \phi(\mathbf{t}) &= -\varkappa_3 \boldsymbol{\beta} + \frac{1}{\sigma} v^{1-\sigma} \mu \mathbf{n}, \\ \phi(\mathbf{n}) &= -\mu \frac{1}{\sigma} v^{1-\sigma} \mathbf{t}, \end{aligned} \quad (7)$$

where $\varkappa_3 = \langle \phi(\boldsymbol{\beta}), \mathbf{t} \rangle$ represents a sufficiently smooth potential. Moreover, the magnetic vector field \mathbf{V} is described by

$$\mathbf{V} = \frac{1}{\sigma} v^{1-\sigma} \mu \boldsymbol{\beta} + \varkappa_3 \mathbf{n}.$$

Hausdorff energies of Lorentz forces of spherical magnetic curves

In this part, we investigated Hausdorff energies of normalized and recursional magnetic fields of timelike magnetic curves associated with the Bishop system in space.

β –magnetic curves

★ The Hausdorff derivatives of ϕ Lorentz force for a spherical β –magnetic curve associated with a spherical system are obtained by

$$\begin{aligned}\mathcal{H}\mathcal{D}^{\star}\phi(\beta) &= \frac{1}{\sigma^2}\mu v^{2-2\sigma}\mathbf{n} \\ &\quad + (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\mathbf{t} - v^{-2\sigma+2}\frac{1}{\sigma^2}\beta, \\ \mathcal{H}\mathcal{D}^{\star}\phi(\mathbf{t}) &= (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\beta \\ &\quad + v^{1-\sigma}\frac{1}{\sigma}(\mathcal{H}\mathcal{D}^{\star}\varkappa_1)\mathbf{n} - \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1\mu + \frac{1}{\sigma^2}v^{2-2\sigma}\right)\mathbf{t}, \\ \mathcal{H}\mathcal{D}^{\star}\phi(\mathbf{n}) &= -\frac{1}{\sigma}v^{1-\sigma}(\mathcal{H}\mathcal{D}^{\star}\varkappa_1)\mathbf{t} \\ &\quad - \frac{1}{\sigma}\varkappa_1v^{1-\sigma}\mu\mathbf{n} + \varkappa_1v^{1-\sigma}\frac{1}{\sigma}\beta, \\ \mathcal{H}\mathcal{D}^{\star}\mathbf{V} &= v^{1-\sigma}\frac{1}{\sigma}(\mathcal{H}\mathcal{D}^{\star}\varkappa_1)\beta \\ &\quad + \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1 - \frac{1}{\sigma}v^{1-\sigma}\mu\right)\mathbf{t}.\end{aligned}$$

★ The Hausdorff energies of ϕ Lorentz force of a β –magnetic curve under the spherical system are described as follows:

$$\begin{aligned}\text{energy}(\phi(\beta)) &= \frac{1}{2}\int_{\sigma}\left(1 + \left(v^{-2\sigma+2}\frac{1}{\sigma^2}\mu\right)^2\right. \\ &\quad \left.+ \left((1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + \left(v^{-2\sigma+2}\frac{1}{\sigma^2}\right)^2\right)dv, \\ \text{energy}(\phi(\mathbf{t})) &= \frac{1}{2}\int_{\sigma}\left(1 + \frac{1}{\sigma^2}(\mathcal{H}\mathcal{D}^{\star}\varkappa_1)^2v^{2-2\sigma}\right. \\ &\quad \left.+ \left((1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1\mu + \frac{1}{\sigma^2}v^{2-2\sigma}\right)^2\right)dv, \\ \text{energy}(\phi(\mathbf{n})) &= \frac{1}{2}\int_{\sigma}\left(1 + v^{2-2\sigma}(\mathcal{H}\mathcal{D}^{\star}\varkappa_1)^2\frac{1}{\sigma^2}\right. \\ &\quad \left.+ v^{2-2\sigma}\varkappa_1^2\frac{1}{\sigma^2} + \varkappa_1^2\frac{1}{\sigma^2}\mu^2v^{2-2\sigma}\right)dv, \\ \text{energy}(\mathbf{V}) &= \frac{1}{2}\int_{\sigma}\left(1 + \frac{1}{\sigma^2}v^{2-2\sigma}(\mathcal{H}\mathcal{D}^{\star}\varkappa_1)^2\right. \\ &\quad \left.+ \left(\frac{1}{\sigma}v^{1-\sigma}\varkappa_1 - \frac{1}{\sigma}v^{1-\sigma}\mu\right)^2\right)dv.\end{aligned}$$

\mathbf{t} –magnetic curves

★ The Hausdorff derivatives of ϕ Lorentz force for a spherical \mathbf{t} –magnetic curve associated with a spherical system are given by

$$\begin{aligned}
 {}_{\mathcal{H}}\mathcal{D}^{\star}\phi(\beta) &= -v^{-2\sigma+2}\frac{1}{\sigma^2}\beta + \left(-\frac{1}{\sigma}\mu v^{1-\sigma}\varkappa_2\right. \\
 &\quad \left.+ (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)\mathbf{t} + \left(v^{2-2\sigma}\mu\frac{1}{\sigma^2} + \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_2)\right)\mathbf{n}, \\
 {}_{\mathcal{H}}\mathcal{D}^{\star}\phi(\mathbf{t}) &= (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\beta - \left(\frac{1}{\sigma^2}\mu^2v^{-2\sigma+2}\right. \\
 &\quad \left.+ v^{2-2\sigma}\frac{1}{\sigma^2}\right)\mathbf{t} + \left(v^{2-2\sigma}\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu) + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)\mathbf{n}, \\
 {}_{\mathcal{H}}\mathcal{D}^{\star}\phi(\mathbf{n}) &= \left(\mu\frac{1}{\sigma^2}v^{2-2\sigma} - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_2)\right)\beta \\
 &\quad - \left(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu)v^{2-2\sigma} + \frac{1}{\sigma}\varkappa_2v^{1-\sigma} + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)\mathbf{t} - \frac{1}{\sigma^2}\mu^2v^{2-2\sigma}\mathbf{n}, \\
 {}_{\mathcal{H}}\mathcal{D}^{\star}\mathbf{V} &= \left(\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu) + \frac{1}{\sigma}v^{1-\sigma}\varkappa_2\right)\beta \\
 &\quad - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_2)\mathbf{t} - \frac{1}{\sigma}v^{1-\sigma}\varkappa_2\mu\mathbf{n}.
 \end{aligned}$$

★ The Hausdorff energies of ϕ Lorentz force of a \mathbf{t} -magnetic curve under the spherical system are obtained as follows:

$$\begin{aligned}
 \text{energy}(\phi(\beta)) &= \frac{1}{2}\int_{\sigma} \left(1 + \left(\frac{1}{\sigma^2}v^{2-2\sigma}\right)^2 + \left(-\frac{1}{\sigma}\mu v^{1-\sigma}\varkappa_2\right.\right. \\
 &\quad \left.\left.+ (1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + \left(\frac{1}{\sigma^2}\mu v^{2-2\sigma} + \frac{1}{\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_2)v^{1-\sigma}\right)^2\right) dv, \\
 \text{energy}(\phi(\mathbf{t})) &= \frac{1}{2}\int_{\sigma} \left(1 + \left(\frac{1}{\sigma^2}\mu^2v^{-2\sigma+2} + v^{2-2\sigma}\frac{1}{\sigma^2}\right)^2\right. \\
 &\quad \left.\left((1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + \left(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu)v^{2-2\sigma}\right)^2\right) dv, \\
 \text{energy}(\phi(\mathbf{n})) &= \frac{1}{2}\int_{\sigma} \left(1 + \left(\mu\frac{1}{\sigma^2}v^{2-2\sigma} - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_2)\right)^2\right. \\
 &\quad \left.+ \left(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu)v^{2-2\sigma} + \frac{1}{\sigma}\varkappa_2v^{1-\sigma} + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)^2 + (v^{2-2\sigma}\frac{1}{\sigma^2}\mu^2)^2\right) dv, \\
 \text{energy}(\mathbf{V}) &= \frac{1}{2}\int_{\sigma} \left(1 + \left(\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu) + \frac{1}{\sigma}v^{1-\sigma}\varkappa_2\right)^2\right. \\
 &\quad \left.+ \frac{1}{\sigma^2}v^{2-2\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_2)^2 + \mu^2\frac{1}{\sigma^2}v^{2-2\sigma}\varkappa_2^2\right) dv.
 \end{aligned}$$

\mathbf{n} -magnetic curves

★ The Hausdorff derivatives of ϕ Lorentz force for a spherical \mathbf{n} -magnetic curve associated with a spherical system are obtained by

$$\begin{aligned}
 {}_{\mathcal{H}}\mathcal{D}^{\star}\phi(\beta) &= \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_3)\mathbf{t} \\
 &\quad + \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\mu\mathbf{n} - \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\beta, \\
 {}_{\mathcal{H}}\mathcal{D}^{\star}\phi(\mathbf{t}) &= \left(\frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu) + \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)\mathbf{n} \\
 &\quad - \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_3)\beta - \left(\frac{1}{\sigma^2}v^{2-2\sigma}\mu^2 + \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\right)\mathbf{t}, \\
 {}_{\mathcal{H}}\mathcal{D}^{\star}\phi(\mathbf{n}) &= v^{2-2\sigma}\mu\frac{1}{\sigma^2}\beta - \left(\frac{1}{\sigma^2}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu)v^{2-2\sigma}\right. \\
 &\quad \left.+ \mu(1-\sigma)\frac{1}{\sigma}v^{1-2\sigma}\right)\mathbf{t} - \frac{1}{\sigma^2}\mu^2v^{2-2\sigma}\mathbf{n}, \\
 {}_{\mathcal{H}}\mathcal{D}^{\star}\mathbf{V} &= \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\mu)\beta + \left(\frac{1}{\sigma^2}v^{2-2\sigma}\mu\right. \\
 &\quad \left.- \frac{1}{\sigma}v^{1-\sigma}\varkappa_3\mu\right)\mathbf{t} + \frac{1}{\sigma}v^{1-\sigma}({}_{\mathcal{H}}\mathcal{D}^{\star}\varkappa_3)\mathbf{n}.
 \end{aligned}$$

★ The Hausdorff energies of ϕ Lorentz force of a \mathbf{n} -magnetic curve under the spherical system are constructed as follows:

$$\begin{aligned}
\text{energy}(\phi(\beta)) &= \frac{1}{2} \int_{\sigma} \left(1 + \left(\frac{1}{\sigma} v^{1-\sigma} (\mathcal{H}\mathcal{D}^{\star} \kappa_3) \right)^2 \right. \\
&\quad \left. + \frac{1}{\sigma^2} v^{2-2\sigma} \kappa_3^2 \mu^2 + \frac{1}{\sigma^2} v^{2-2\sigma} \kappa_3^2 \right) dv, \\
\text{energy}(\phi(t)) &= \frac{1}{2} \int_{\sigma} \left(1 + \left(\frac{1}{\sigma} v^{1-\sigma} (\mathcal{H}\mathcal{D}^{\star} \mu) + \mu(1-\sigma) \frac{1}{\sigma} v^{1-2\sigma} \right)^2 \right. \\
&\quad \left. + \frac{1}{\sigma^2} v^{2-2\sigma} (\mathcal{H}\mathcal{D}^{\star} \kappa_3)^2 + \left(\frac{1}{\sigma^2} v^{2-2\sigma} \mu^2 + \frac{1}{\sigma} v^{1-\sigma} \kappa_3 \right)^2 \right) dv, \\
\text{energy}(\phi(n)) &= \frac{1}{2} \int_{\sigma} \left(1 + \left(\frac{1}{\sigma^2} \mu v^{-2\sigma+2} \right)^2 \right. \\
&\quad \left. + \left(\frac{1}{\sigma^2} (\mathcal{H}\mathcal{D}^{\star} \mu) v^{2-2\sigma} + \mu(1-\sigma) \frac{1}{\sigma} v^{1-2\sigma} \right)^2 + \left(\frac{1}{\sigma^2} \mu^2 v^{-2\sigma+2} \right)^2 \right) dv, \\
\text{energy}(V) &= \frac{1}{2} \int_{\sigma} \left(1 + \frac{1}{\sigma^2} v^{2-2\sigma} (\mathcal{H}\mathcal{D}^{\star} \mu)^2 \right. \\
&\quad \left. + \left(\frac{1}{\sigma} v^{1-\sigma} \mu - \frac{1}{\sigma} v^{1-\sigma} \kappa_3 \mu \right)^2 + \frac{1}{\sigma^2} v^{2-2\sigma} (\mathcal{H}\mathcal{D}^{\star} \kappa_3)^2 \right) dv.
\end{aligned}$$

Applications for energy density peaks

The graphical and energy analysis of n -magnetic curve is presented in Figs. 1, 2, 3 and 4, providing insights into the performance of Hausdorff energy density peaks in conformable model.

Conclusions

In this article, an approach for spherical magnetic curves under the spherical system in space are given. Firstly, the Hausdorff derivative of the Lorentz spherical magnetic fields $\phi(\beta)$, $\phi(t)$, $\phi(n)$ of spherical magnetic curves is constructed. On the other hand, the Lorentz spherical magnetic fields by considering Hausdorff derivative definition are presented. Eventually, the Hausdorff energies of these spherical Lorentz magnetic fields $\phi(\beta)$, $\phi(t)$, $\phi(n)$ associated with the spherical system in S^2 spherical space are computed.

Data availability

Data will be provided by corresponding author on reasonable request.

Received: 30 July 2024; Accepted: 24 October 2024

Published online: 04 February 2025

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Author contributions

All authors reviewed the manuscript.

Declarations

Competing interests

The authors declare no competing interests.

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