

Deformation effects on isospin mixing and isobar analogue resonance for $^{74-80}\text{Kr}$ isotopes

Research Article

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Abstract:

Pyatov's method has been applied to investigate Fermi beta transitions in deformed $^{74-80}\text{Kr}$ isotopes. This self-consistent method, which was used to study the isobar analogue states in the spherical odd-odd nuclei, has to date not been applied for the isobar analogue states in deformed nuclei. The nucleon-nucleon residual interaction has been included so that the broken isospin symmetry in the mean field approximation has been restored and the strength parameter of the effective interaction has been taken out to be a free parameter. The energies and wave functions of the isobaric analogue excitations in $^{74-80}\text{Rb}$ isotopes have been obtained within the framework of the pnQRPA method. The probability of the isospin mixing in the ground states and the centroid energies of the isobar analogue resonance have been presented and the deformation effects on these quantities have been quantified.

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1. Introduction

The problem of isospin mixing in nuclear ground states is of great importance in the experimental determination of the vector coupling constant of nucleon β -decay, as well as in the description of the isobar analogue state energies and isospin multiples [1–4]. Isospin mixing is basi-

cally caused by the Coulomb potential. Coulomb forces are more dominant in nuclei for which the proton numbers and the neutron numbers are close to each other. Therefore, the probability of isospin mixing in ground states of proton-rich nuclei is greater when compared with the other nuclei.

Isospin mixing in the nuclear ground state was studied in various models: A two liquid hydrodynamic model was used to estimate the energies of the collective isovector monopole excitation with an isospin of $T = T_0 + 1$ [5]. The admixture of this excited state with the ground state

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(isospin of $T = T_0$) resulting from the Coulomb potential, turned out to be small $\approx(0.1-0.3)\%$ for all stable nuclei with $A > 40$. Quantitative estimates performed by using the shell model [6–8] are approximately an order of magnitude larger than the estimate of Bohr and Mottelson. There are several factors that account for such a difference in the mentioned estimates. Firstly, the shell model calculations were performed in the particle-hole approximation and used a limited number of configurations. Secondly, the residual isovector interaction was either neglected or included in the Tamm-Dankoff approximation. Thirdly, the residual interaction was not related to the shell model potential in a self-consistent way. Isospin mixing for the proton-rich nuclei between $A=80$ and $A=100$ was calculated in previous works [9–19]. The Hartree-Fock calculations for these nuclei predicted isospin mixing of the order of 3–5% [12]. With the inclusion of the random phase approximation in the calculations, isospin mixing increased by amount of 15–20% [9].

The effect of nuclear deformation on the isospin mixing matrix elements was investigated for deformed configurations in ^{18}F [20]. An enhancement of the mixing with deformation was found. The problem of isospin mixing is closely related to isospin symmetry breaking. It is well known that the nuclear shell model breaks the symmetries of the total nucleus Hamiltonian [21]. The best known example of this is the violation of the rotational invariance in deformed nuclei. The self-consistent deformed mean field approximation is a suitable basis for deformed nuclei, but the symmetry violations in the shell model basis are also valid in the self-consistent basis.

The probability of isospin mixing in the nuclear ground states is small for the neutron rich nuclei. However, isospin symmetry breaking leads to an increase in the isospin admixture probability. Hence, the symmetry violations stemming from the mean field approximations must be restored for neutron rich nuclei. The schematic residual $(\tau_1 \cdot \tau_2)$ effective interaction was included in the self-consistent deformed mean field basis for $^{70-78}\text{Kr}$ isotopes [22]. In order to understand the effect of the symmetry breaking, the isospin mixing and Fermi transitions were investigated with and without Coulomb interaction. The consideration of the residual $(\tau_1 \cdot \tau_2)$ effective interaction leads to a decrease in the isospin admixture probability. Hence, the residual $(\tau_1 \cdot \tau_2)$ force can be called the isospin restoring force.

Investigation of isospin mixing and Fermi transitions with and without Coulomb interaction is a suitable way to visualise the isospin symmetry breaking effects, but the exclusion of the Coulomb interaction is not physically correct in restoring isospin symmetry breaking. Thus, a more realistic approximation should be used to restore the sym-

metry violation. The isospin violation in the mean field level for the neutron excess nuclei has been eliminated within nuclear density functional theory [23]. A significant dependence of the magnitude of isospin breaking on the parametrization of the nuclear interaction term has been found by the Warsaw group [23]. Hence, the nucleon-nucleon residual interaction potential should be included in such a way that the isospin symmetry violation in the mean field level is restored. Pyatov's method is an efficient way for the restoration of this symmetry violation [24]. This method was originally introduced to restore the broken Galilean invariance of the pairing interaction [24, 25]. Later, the method was applied for the restoration of translational invariance [26] and rotational invariance [27]. These restorations played an important role in the investigation of the electric dipole excitations (1^-) and magnetic dipole excitations (1^+) in even-even nuclei. The same method was used to restore the isospin invariance of the nuclear Hamiltonian and the problem of the isospin mixing in the ground states was studied for the spherical odd-odd nuclei [28–31]. The method has not been applied for the study of the isobar analogue states in the deformed odd-odd nuclei yet.

We aim to study the deformation effects on the isospin impurity in the ground states and the isobar analogue states. As mentioned above, the main reason for the isospin impurity is the electromagnetic interactions between protons. However, the probability for the isospin mixing in the ground states increases in the mean field approximation. This case stems from the violation of the rotational invariance in isospin space due to the mean field approximation. The conservation of the other quantities such as angular momentum and particle number has no influence on the isospin mixing. For instance, the conservation of angular momentum is important in the study of magnetic dipole excitations and hence, the rotational invariance in ordinary space must be restored to investigate M1 transitions. Pyatov's method is used to eliminate this symmetry violation stemming from the mean field approximation. The present calculations show the dependence of the isospin mixing and the isobar analogue states on the deformation parameter within the framework of pnQRPA formalism. Moreover, a comparison of the isospin mixing and the isobar analogue states obtained according to the calculated deformations with those obtained from experimental deformations is presented in this work. Let us emphasize that we do not aim to make a comparison between the calculated and experimental deformations, the aim of the present work is to merely see the effect of the different deformations on the isospin mixing and the isobar analogue states.

2. Formalism

The conserved quantities such as linear momentum, angular momentum and particle number are a consequence of the invariance of the total nucleus Hamiltonian under symmetry transformations. However, in constructing a nuclear model or obtaining an approximate solution to a problem, Hamiltonians with broken symmetry are often involved. For instance, the study of the 1^- states in even-even nuclei (electric dipole excitations) is related to the translational invariance of the total Hamiltonian. The restoration of the rotational invariance in coordinate space is important in the investigation of the 1^+ states in even-even nuclei (magnetic dipole excitations). Unlike the rotational invariance in coordinate space, the rotational invariance in isospin space is not an exact symmetry of the total Hamiltonian. However, the rotational invariance in isospin space is an exact symmetry of the nuclear part of the total Hamiltonian. In other words, the nuclear part of the total Hamiltonian commutes with the isospin operator. This commutativity is violated in the mean field approximation and its restoration plays an important role in understanding the isobaric analogue excitations. The Fermi β -decay operators are described as

$$T^+ = \sum_{i=1}^A t_+(i), \quad (1)$$

$$T^- = [T^+]^\dagger = \sum_{i=1}^A t_-(i),$$

where t_+ and t_- are isospin raising and lowering operators, respectively. Let us define a new charge exchange operator as below:

$$F^\rho = \frac{1}{2}(T^+ + \rho T^-) \quad (\rho = \pm). \quad (2)$$

The single quasi particle Hamiltonian is defined as follows:

$$H_{sqp} = \sum_{s\sigma} \varepsilon_s(\tau) \alpha_{s\sigma}^\dagger(\tau) \alpha_{s\sigma}(\tau) \quad (\tau = n, p), \quad (3)$$

where $\varepsilon_s(\tau)$ is the single quasi-particle (sqp) energy, and the $\alpha_{s\sigma}^\dagger(\tau)$ ($\alpha_{s\sigma}(\tau)$) is the quasi-particle creation (annihilation) operator. Let us note that n and p abbreviations are used instead of the usual s_n and s_p , respectively. The nuclear part of the total Hamiltonian must commute with Fermi β -decay operator

$$[H - V_c, F^\rho] = 0. \quad (4)$$

This commutativity is however broken in the mean field approximation as described below

$$[H_{sqp} - V_c, F^\rho] = \sum_{np} E_{np}^\rho (C_{np}^\dagger - \rho C_{np}) \neq 0, \quad (5)$$

where, V_c is the Coulomb's term of a deformed Woods-Saxon potential [32]. The reduced matrix element and quasi boson operators in Eq. (5) are given as follows:

$$E_{np}^\rho = \frac{1}{2} [\varepsilon_{np} (\overline{b_{np}} - \rho b_{np}) + \overline{d_{np}} + \rho d_{np}],$$

where $\varepsilon_{np} = \varepsilon_n + \varepsilon_p$ is the sum of the single quasi neutron and quasi proton energies,

$$\overline{b_{np}} = \langle s_n s_p \rangle u_n v_p$$

$$\overline{d_{np}} = \langle s_n V_c(r) s_p \rangle u_n v_p$$

$$(\text{Note : } X_{np} = \frac{\overline{X_{np}}}{u_n v_p} u_p v_n, \quad X = b, d)$$

$$C_{np}^\dagger = \frac{1}{\sqrt{2}} \sum_{\sigma_n, \sigma_p} (-1)^{s_p - \sigma_p} \alpha_{s_n \sigma_n}^\dagger \alpha_{s_p - \sigma_p}^\dagger$$

$$C_{np} = [C_{np}^\dagger]^\dagger.$$

The isospin invariance of the nuclear part violated in the mean field level of approximation can be restored by the inclusion of an effective interaction potential defined as

$$h = \sum_{\rho=\pm} \frac{1}{4\gamma_\rho} [H_{sqp} - V_c, F^\rho]^\dagger [H_{sqp} - V_c, F^\rho]. \quad (6)$$

The effective interaction constant is obtained from the following condition

$$[H_{sqp} - V_c + h, F^\rho] = 0 \quad (7)$$

and taken out to be a free parameter.

$$\gamma_\rho = \langle 0 | [[H_{sqp} - V_c, F^\rho], F^\rho] | 0 \rangle. \quad (8)$$

The pnQRPA Hamiltonian for the investigation of the isobar analogue states has been described as follows:

$$H = H_{sqp} + h. \quad (9)$$

The phonon creation operator for the isobar analogue states is defined by:

$$|i\rangle = Q_i^\dagger |0\rangle = \sum_{np} [\psi_{np}^i C_{np}^\dagger + \varphi_{np}^i C_{np}] |0\rangle, \quad (10)$$

where ψ_{np}^i and φ_{np}^i are the forward and backward amplitudes, respectively. Solving the following equation provides the energies and wave functions of the isobar analogue excitations:

$$[H_{sqp} + h, Q_i^\dagger] |0\rangle = \omega_i Q_i^\dagger |0\rangle, \quad (11)$$

where ω_i corresponds to the excitation energies for the isobaric analogue states. The following linear equations are obtained for the ψ_{np}^i and φ_{np}^i amplitudes:

$$\sum_{np} [(\rho_{n'p'np} - \omega_i \delta_{n'n} \delta_{p'p}) \psi_{np}^i + \eta_{n'p'np} \varphi_{np}^i] = 0$$

$$\sum_{np} [\eta_{n'p'np} \psi_{np}^i + (\rho_{n'p'np} - \omega_i \delta_{n'n} \delta_{p'p}) \varphi_{np}^i] = 0. \quad (12)$$

In order to calculate the QRPA matrices, the double commutators described below are solved.

$$\rho_{n'p'np} = [C_{n'p'}, [H, C_{np}^\dagger]] \quad (13)$$

$$\eta_{n'p'np} = [C_{n'p'}, [H, C_{np}]]. \quad (14)$$

Matrix expressions for these are:

$$\rho_{n'p'np} = \varepsilon_{np} \delta_{n'n} \delta_{p'p} + \frac{E_{np}^+ E_{n'p'}^+}{2\gamma_+} + \frac{E_{np}^- E_{n'p'}^-}{2\gamma_-}$$

$$\eta_{n'p'np} = \frac{E_{np}^+ E_{n'p'}^+}{2\gamma_+} - \frac{E_{np}^- E_{n'p'}^-}{2\gamma_-}.$$

The excitation energies (ω_i) are the roots of the following equation:

$$\sum (\rho_{n'p'np} - \omega_i \delta_{n'n} \delta_{p'p}) \cdot \sum (\rho_{n'p'np} - \omega_i \delta_{n'n} \delta_{p'p}) - \sum \eta_{n'p'np} \cdot \sum \eta_{n'p'np} = 0.$$

Total β decay strength for the Fermi transitions is given as follows:

$$S^{\beta^\pm} = \sum_i |M_{\beta^\pm}^i(0_{g.s.}^+ \rightarrow 0_i^+)|^2 \quad (15)$$

$$M_{\beta^\pm}^i(0_{g.s.}^+ \rightarrow 0_i^+) = \langle 0 | [Q_i, T^\pm] | 0 \rangle.$$

These strengths must comply with the sum rule in the following form:

$$S^{\beta^-} - S^{\beta^+} = N - Z. \quad (16)$$

The centroid energy of the isobar analogue states and the probability of the isospin mixing in the ground state of the parent nucleus have been calculated by using Eq. (17) and (18), respectively:

$$\omega_{IAS} = \frac{\sum \omega_i (|M_{\beta^-}^i|^2 - |M_{\beta^+}^i|^2)}{\sum (|M_{\beta^-}^i|^2 - |M_{\beta^+}^i|^2)} \quad (17)$$

$$b^2 = \frac{1}{2(T_0 + 1)} \sum_i |M_{\beta^+}^i|^2 \quad (18)$$

$$T_0 = (N - Z)/2.$$

Rotational invariance in isospin space is not an exact symmetry due to Coulomb interaction. In fact, the variable part of Coulomb potential is responsible for the symmetry breaking. Namely, the isospin invariance would be an exact symmetry if the Coulomb potential became zero or remained constant with nuclear volume. Thus, the isobar analogue resonance would appear at a constant energy and have all β decay strengths. It can be said that the probability of isospin mixing in nuclear ground states would thus be zero. Also, the isobar analogue resonance would appear at zero energy in the case of no Coulomb interaction.

3. Results and discussions

The isobaric analogue excitations in $^{74-80}\text{Rb}$ isotopes have been investigated within the framework of the pnQRPA method. Energy distribution of the 0^+ excited states has been calculated for different values of the deformation parameter. Variations of isobar analogue resonance and

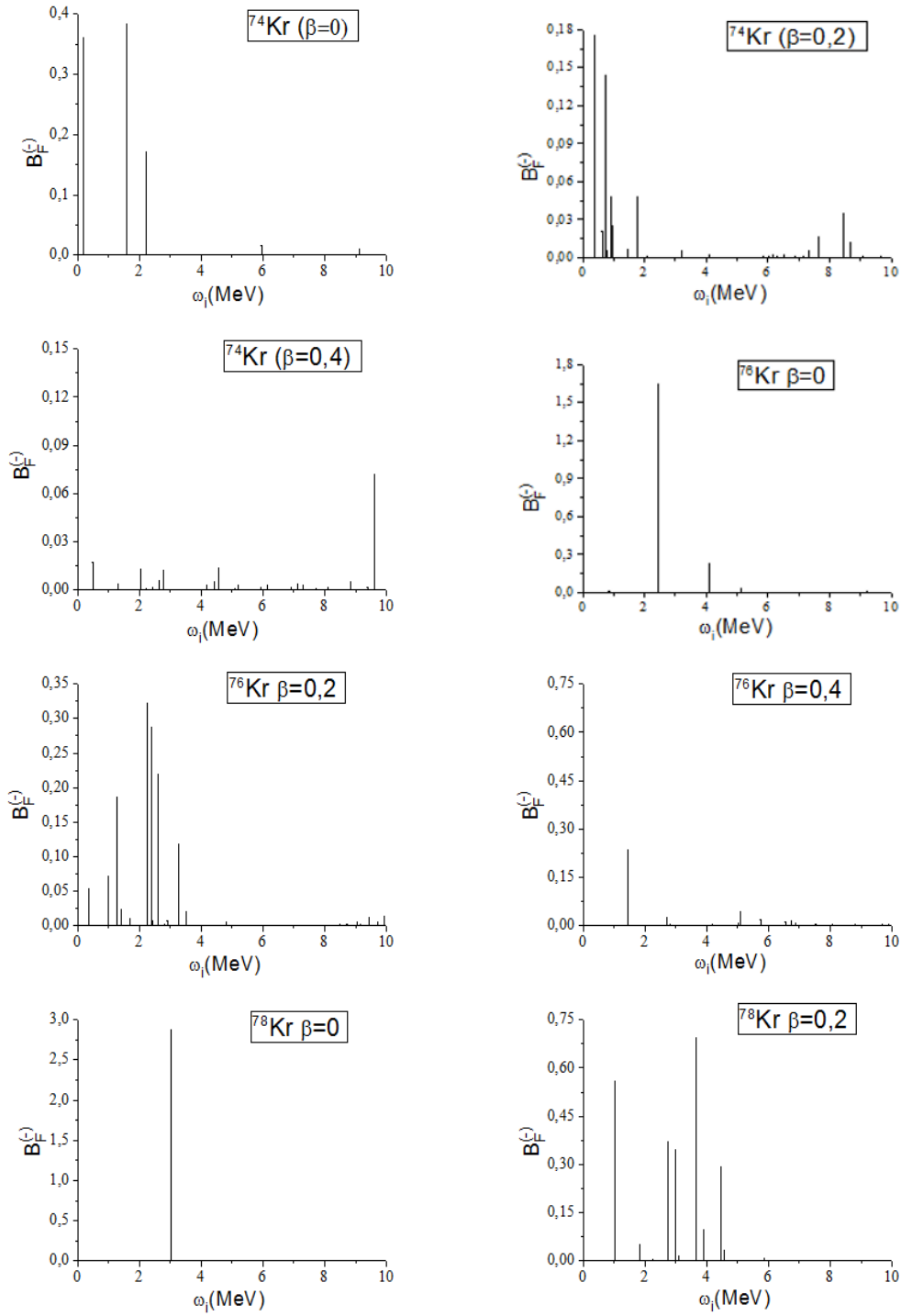


Figure 1. The energy spectrum of the β^- transition strength for $A=74, 76, 78$ and 80 (top-to-bottom) isotopes. The excitation energies have been obtained over the ground state of the daughter nucleus.

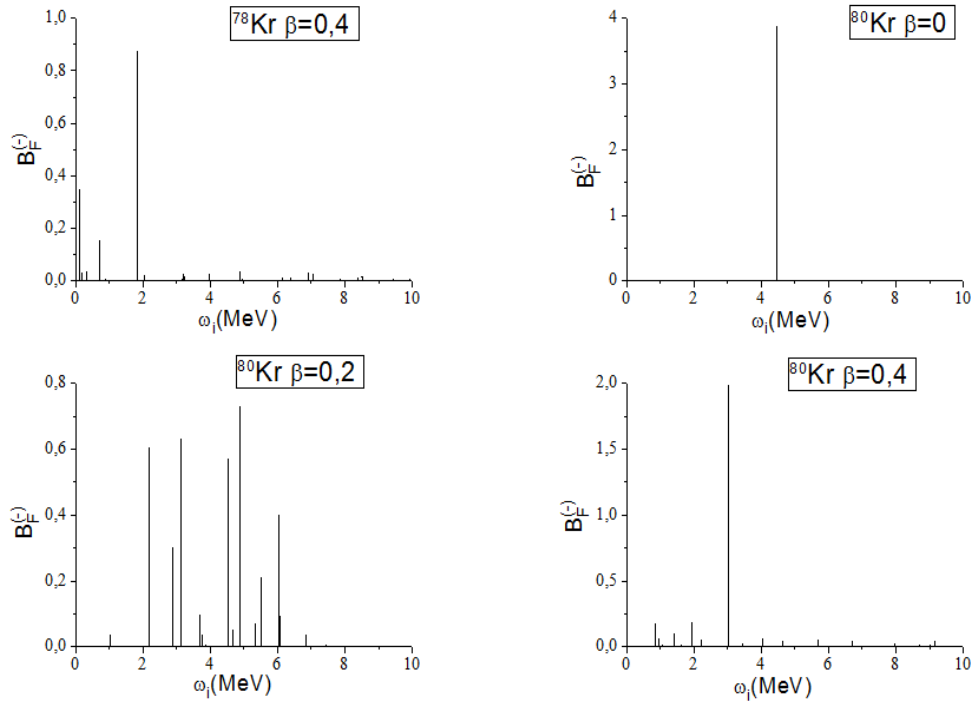


Figure 1. Figure 1 (cont.)

isospin mixing by deformation have been obtained for different isotopes. Naturally, the calculated results depend on the choice of the mean field parameters and pairing gaps. The sensitivity of isobar analogue states to the mean field parameters and pairing interaction was studied by using Pyatov's method within the pnQRPA formalism [28, 33]. In order to understand the deformation effects on the isobar analogue states in $^{74-80}\text{Rb}$ isotopes, the mean field parameters and pairing gaps are fixed. The Nilsson single particle energies and wave functions have been calculated with the deformed Woods-Saxon potential [32, 34]. All energy levels up to 8 MeV have been considered for neutrons and protons, and the proton and neutron pairing gaps have been determined as $C_n = C_p = 12/\sqrt{A}$ MeV.

The beta strength distribution for different deformations are presented in Figure 1. In case of a deformed basis, the β^- transition strength shows more fragmentation over the final states, which can be attributed to the decrease of the degeneracy with deformation. The consideration of deformation effects in a single particle basis leads to extra violation in the isospin invariance of the nuclear Hamiltonian. Hence, this extra-symmetry violation must be compensated for by the inclusion of the effective interaction potential. The variation of the strength parameter of the effective interaction potential by deformation is shown in Figure 2. It should be emphasized that the effective inter-

action potential is inversely proportional to its strength parameter (see Eq. (6)). The decrease in the γ_- parameter with the increase of the deformation parameter verifies the importance of the residual interactions between nucleons in the deformed structure. The increase in the γ_+ parameter (in absolute value) with deformation is due to the effect of the reduced matrix element (E_{np}^+) on the value of the effective interaction potential in Eq. (6). As seen, the effective interaction depends on the multiplication of two commutators which contain the reduced matrix element (E_{np}^p) (see Eq. (5)). Hence, the increase of the reduced matrix element (E_{np}^+) can be more dominant than the increase of the γ_+ parameter and this case can lead to an increase in the effective interaction potential. In heavier isotopes, the symmetry violation can be restored by a smaller effective interaction since the effect of the Coulomb potential is more negligible. Therefore, the effective interaction constant has a larger value in heavier isotopes. It should also be noted that the difference between γ_+ and γ_- stems from the pairing correlations between nucleons. This case is confirmed by the calculations performed for spherical nuclei [35]. The γ_+ and γ_- constants would be equal to each other in case there was no pairing effect.

The existence of the β^+ transitions in neutron excess nuclei ($N > Z$) originates from the fact that the isospin invariance is not an exact symmetry of the total Hamil-

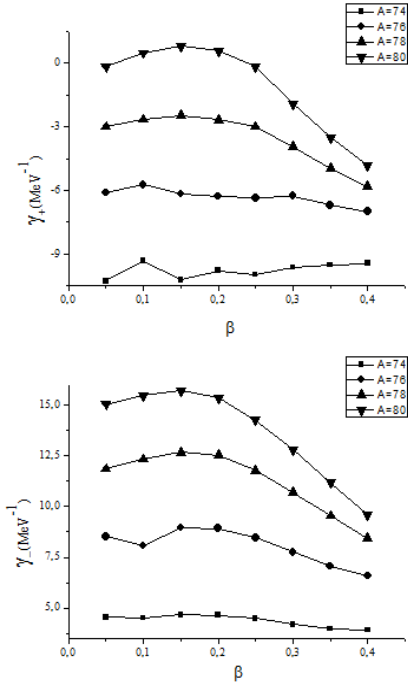


Figure 2. Variation of the effective interaction constant with deformation.

tonian. The β^+ transition probability would be zero if the isospin invariance was an exact symmetry of the total Hamiltonian, all the beta strength would belong to only one degenerate state in neighboring odd-odd ($N-1, Z+1$) nuclei and therefore, there would be no possibility of isospin mixing in the ground states. Deformation effects on the isospin mixing and the centroid energies of the isobar analogue states are shown in Figure 3 and 4, respectively. As seen, the isospin mixing has an increasing tendency with the deformation parameter. This is an expected result due to the extra symmetry violation. However, isospin mixing shows a sudden decrease between $\beta = 0.30$ and $\beta = 0.35$. This sudden decrease can be related to the effect of the variation in the deformation on the single neutron and proton levels. A small variation in the deformation parameter can lead to a sudden exchange of any two single-particle levels. This case is also confirmed by the single-proton and neutron levels of the Nilsson potential [36]. As known, the influence of the deformation on a single particle level increases with the increase of the orbital quantum number (ℓ). While the ground state of the neighboring nuclei for $\beta = 0.30$ is dominated by the $f_{7/2} \rightarrow f_{7/2}$ configuration, the dominant contribution to the ground state for $\beta = 0.35$ comes from the $s_{1/2} \rightarrow s_{1/2}$ configuration. The deformation effect on the isospin symmetry-breaking for $\beta = 0.35$ is negligible

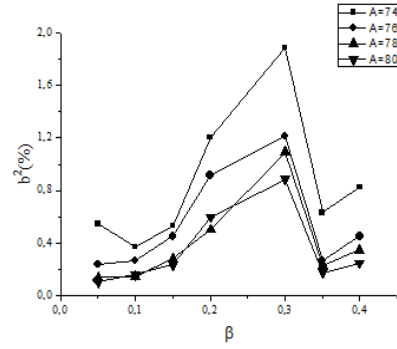


Figure 3. Variation of the isospin mixing with deformation.

due to the structure of the ground state. This case leads to a sudden decrease in the isospin admixture probability. There are two different curves for each isotope in Figure 4. The first curve (squares) shows the average energies obtained from Eq. (17), whereas the second one (circles) shows the resonance energies observed in the spectrum. The variation of deformation leads to shifting of the total decay strength to the other energy regions. Hence, the resonance energy observed in the spectrum may exhibit abrupt changes with the variation of the deformation parameter. However, the average energy of the isobar analogue states exhibits a more stable variation with the deformation. The average energy usually has a decreasing tendency with the increase of the deformation parameter. Therefore, it can be said that the total decay strength shifts to the low-energy region of the spectrum in which the β^+ transition strength is mostly collectivized. The general increasing tendency of the isospin admixture probability (which is directly proportional to the β^+ decay strength) is consistent with the variation of the average energy.

The calculated deformations and the corresponding residual interaction parameters, isospin admixture probabilities and centroid energies are presented in Table 1. The same quantities have also been computed by using the experimental deformations [37] which are obtained from the experimental quadrupole decay probabilities and the calculated results are shown in Table 2. The present deformations in Table 1 have been determined from the single particle energies as shown in [36] and the calculated values of the residual interaction parameters and average energies are not very far off from those presented in Table 2. However, the calculated values for the isospin admixture probability by using two different deformations are remarkably different from each other. This is an expected result due to the sensitivity of the isospin mixing to the deformation in nuclear structure (see Fig. 3).

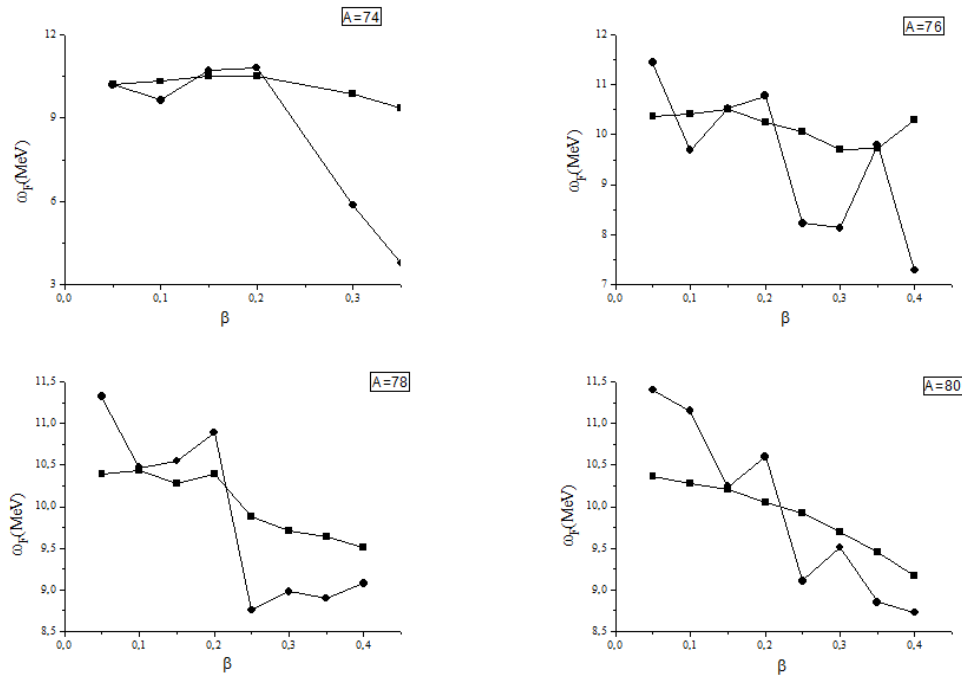


Figure 4. Variation of the centroid energy obtained over the ground state of the parent nucleus with deformation.

Table 1. Calculated effective interaction parameter, isospin admixture probability and centroid energy values corresponding to the calculated deformations.

Isotopes	β	γ_+ (MeV ⁻¹)	γ_- (MeV ⁻¹)	b^2 (%)	ω_{IAS} (MeV)
⁷⁴ Kr	0.05	-10.29	4.55	0.547	10.21
⁷⁶ Kr	0.01	-5.86	6.88	0.266	10.62
⁷⁸ Kr	0.01	-3.27	9.99	0.137	10.54
⁸⁰ Kr	0.08	0.24	15.29	0.138	10.34

Table 2. Calculated effective interaction parameter, isospin admixture probability and centroid energy values corresponding to experimental deformations in Ref. [37].

Isotopes	β	γ_+ (MeV ⁻¹)	γ_- (MeV ⁻¹)	b^2 (%)	ω_{IAS} (MeV)
⁷⁴ Kr	0.346	-9.52	3.99	0.606	9.34
⁷⁶ Kr	0.369	-6.99	6.84	0.313	9.82
⁷⁸ Kr	0.303	-4.01	10.61	1.104	9.70
⁸⁰ Kr	0.228	0.07	14.81	0.652	9.98

4. Conclusions

The deformation effects on the isospin mixing and isobar analogue resonance for various Kr isotopes have been treated within pnQRPA. The residual interactions between nucleons have been defined in such a way that the broken isospin invariance in a deformed single particle basis is

restored and the strength parameter of the residual interaction is obtained in a self-consistent way. In the spherical case ($\beta = 0$), the Fermi strength distribution shows no fragmentation over the final states. A more homogeneous distribution is obtained for the deformed basis. Thus, the increasing tendency of the isospin mixing with the deformation parameter verifies the fragmentation of the isobaric analogue excitations.

As known, the isospin mixing in the ground state stems from Coulomb interactions. The variable part of the Coulomb potential is responsible for the isospin symmetry breaking and this part does not show a remarkable change in the spherically-symmetric case ($\beta = 0$) for the nuclei under consideration. Thus, the isospin admixture probability in the spherically-symmetric basis is smaller than that in the deformed basis. It can be concluded that isospin mixing in the ground state of the deformed system can be attributed to the influence of the deformation on the variable part of the Coulomb potential.

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