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Fuzzy LQR-based control to ensure comfort in HVAC system with two different zones

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ABSTRACT

Heating, ventilation, and air conditioning (HVAC) systems are control systems that ensure indoor temperature and air quality meet desired conditions. In this study, a novel control strategy is proposed for an HVAC system operating under two distinct environmental zones with variable flow rates, addressing control challenges arising from external disturbances such as ambient temperature and humidity changes. In the system design, mathematical models were obtained, including the heat losses of two zones to the outdoor environment, as well as the heat transfer dynamics in the cooling unit, fans, and air ducts. For system control, considering ambient temperature, humidity, and variable flow rate, the required airflow was achieved by controlling the dampers placed in the indoor air inlet ducts. The core novelty of this work lies in the development and comparison of advanced control algorithms, including the Linear Quadratic Regulator (LQR), a Particle Swarm Optimization (PSO)-based LQR, and a newly designed PSO-based Fuzzy LQR (FLQR) controller. Comfort conditions were achieved by cooling the temperatures of two different regions from the ambient temperature to approximately 7 °C. The proposed FLQR controller combines the adaptability of fuzzy logic with the optimization capabilities of PSO to enhance system responsiveness and occupant comfort. Simulation results show that the FLQR method improves comfort performance by 90.4 % for Zone-1 and 88.1 % for Zone-2 compared to conventional LQR. The effectiveness of the proposed method (FLQR) is demonstrated through a comprehensive performance evaluation using Mean Squared Error (MSE) metrics, confirming its potential for intelligent HVAC applications.

Nomenclature

$\dot{m}_{ind,in}$	The mass flow rate entered Zone-1 (kg/s)
$\dot{m}_{z_1 a,in}$	The mass flow rate entered to Zone-1 (kg/s)
$\dot{m}_{z_2 a,in}$	The mass flow rate entered to Zone-2 (kg/s)
$\dot{m}_{sva,out}$	The mass flow rate exists from the safety valve (kg/s)
$\dot{m}_{exha,out}$	The mass flow rate exists from exhaust (kg/s)
Q	Convection and transmission heat (J)
W	Work (J)
U	The internal energy (J)
h_{in}, h_{out}	Specific enthalpy(J/kg)

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C_v	Constant heat (kJ/kgK)
C_p	Constant pressure (kJ/kgK)
T	Inner temperature (^oC)
T_n	Instant temperature (^oC)
T_{n-1}	Vicious circle temperature (^oC)
$T_{cool.air}$	Cool air temperature (^oC)
T_{out}	Outside temperature (^oC)
h_{out}	Convection coefficient for outside-surface (J/m^{2}K (^oC)
h_{in}	Convection coefficient for inner-surface (J/m^{2}K (^oC)
k	Transmission coefficient (J/mK)
L_1	Thickness for Zone-1 (m)
L_2	Thickness for Zone-2 (m)
LQR	Linear Quadratic Regulator
PSO	Particle Swarm Optimization Algorithm
Fuzzy – LQR	Fuzzy Linear Quadratic Regulator

1. Introduction

Today, the rapidly increasing world population, construction, and technological developments increase the demand for energy consumption. Due to the decreasing availability of limited energy resources in the face of increasing demand, the efficient use of energy resources has become critically important. The energy consumed in buildings constitutes a significant portion of the total energy consumption in a country. The ratio of the energy consumed in buildings to the total energy consumed is increasing day by day. As air conditioning systems reduce energy consumption, they lead to lower costs [1]. According to the forecasts, the tendency to spend energy will continue to increase in the coming years [2]. HVAC systems (Heating, Ventilating, and Air Conditioning) are actively used to provide fresh and clean airflow, achieve optimal temperature and humidity conditions, and ensure energy savings in indoor environments. The use of HVAC systems has become increasingly significant for the efficient utilization of energy [3]. One of the critical functions of HVAC systems is to provide appropriate environmental conditions. In general, since people spend most of their time indoors, environmental and environmental conditions affect human health and working performance [4]. Numerous studies have been conducted and continue to be carried out on the modeling and control of HVAC systems.

In their study, Lute and Paassen aimed to control and find solutions for maintaining the indoor temperature of a building through optimal control [6]. Additionally, they worked on a control system that would economically manage the heat supply to the building and predict the required temperature value using various control methods. In their study, Asiedu et al. [7] used the genetic algorithm method to achieve the lowest cost in the HVAC system while designing ventilation ducts. This method ensured economic efficiency in HVAC system design. Moreover, it was determined that this approach provided flexibility, simplicity, and ease of incorporating expert knowledge into the algorithm. Bruant et al. [8] compared fuzzy logic and conventional on-off control methods to minimize energy consumption by meeting thermal comfort conditions and indoor air quality in a control zone according to their needs. The study was conducted in a control zone equipped with split air conditioning. The purpose of selecting the fuzzy controller was to reduce the number of rules for a hierarchical architecture and advance rule techniques. According to the data obtained from their study, the conventional on-off control method failed to meet the thermal comfort conditions and indoor air quality. Additionally, it was found that using the fuzzy control method achieved thermal comfort and indoor air quality with lower energy consumption. Engdahl and Svensson designed a system aimed at achieving an optimally air-conditioned indoor environment with efficient and long-term energy use [9]. To this end, a pressure-controlled variable air volume system within the HVAC system was selected, and solution processes were performed. The flow rate differences between diffusers were calculated, basic equations were formed, and a pressure sensor was installed. It was observed that the upper-level supply air flow rate increased the efficiency of workers in the environment. By utilizing 100 % outdoor air, the supply airflow was increased. It was demonstrated that airflow could be changed in different zones without affecting others. The fan pressure setpoints were optimized to minimize fan power requirements and noise levels.

In their study, Yılmaz et al. [10] modeled a variable air flow air conditioning system for an office building and compared the control methods in this office building. In the model developed for the office building, equations for all zones, cooling, and dehumidification coils, the cooling unit, fan, and ducts were obtained, and sub-models were created. Additionally, the entire model of the variable air volume air-conditioning system was developed. Block diagrams based on the models and control systems were drawn using Matlab/Simulink software. Through the damper opening ratios determined in the study, temperature, humidity values, energy amounts, and comfort conditions in the zones were achieved. Singh et al. [5] carried out a study on fuzzy modeling and control of HVAC systems.

Soygüder and Alli, in their study, conducted the mathematical modeling and control of a variable air volume HVAC system with two different zones, considering external environmental conditions and reference temperatures [11]. Here, it is aimed to cool the outdoor temperature until it reaches the reference temperature. Sub-models were created and heat transfer equations of the cooling unit, fans and ducts were found for the heat losses caused by the convection and transmission of the zones. By calculating the damper opening ratios, the airflow rates and temperature values in the zones also ensured the formation of comfort conditions. The numerical simulation of a variable air volume HVAC system using fuzzy logic and PID control methods was conducted with the Matlab/Simulink software package. It was observed that the graphs obtained by the fuzzy logic control reached the desired reference temperature values more quickly with the damper opening ratios. More errors were observed in the PID control of the model compared to the damper opening ratios and the fuzzy logic control of the desired temperature value.

Table 1
Comparison table of studies in the literature.

References	Proposed control methods	Results/findings
Karkamaz [15], 1998	Finite difference approximation method	The short-term performance of heating and cooling coils was analyzed using a numerical model.
Wang et al. [16], 2000	Mechanical model of centrifugal coolers	The control performance of the chiller dynamics on the HVAC system was determined.
Browne and Bansal [17], 2002	Thermal capacitance approach	The simulation model predictions for vapor compression liquid chillers were found to be within a ±10 % range
Alcalá et al. [18], 2003	Genetic algorithm-based fuzzy controller	It was proven during the experiments that energy consumption decreased significantly.
Kulkarni and Hong [19], 2004	Two-position and PI control methods	In terms of thermal comfort, PI control showed much better performance than two-position control.
Yao et al. [20], 2004	Classical control theory	In some cases, errors occurred in simulations while analyzing the dynamic heat transfer of the coils.
Ruano et al. [21], 2006	Control with radial basis neural networks using genetic algorithms	An optimal indoor temperature was achieved, reducing energy consumption.
Nassif et al. [22], 2008	HVAC model with self-tuning parameters via genetic algorithms	The accuracy of measured and predicted results was corrected using the proposed method.
Zhang et al. [23], 2009	Partial differentiation equation (PDE)	The model accurately predicted cooling transitions But overlooked the compressor dynamics by assuming the compression process was polytropic and static.
Platt et al. [24], 2010	Prediction techniques for HVAC systems and genetic algorithm-based adaptive model for optimization	The modeling and prediction results indicated that the shorter the prediction time, the more accurate the indoor temperature estimation.
Nassif et al. [25], 2011	Multi-objective genetic algorithm	The studies showed that energy savings of 16 % could be achieved in the HVAC system. It was concluded that applying a dual-objective optimization problem provided more energy savings compared to a single-objective optimization problem.
Privara et al. [26], 2011	Model Predictive Control (MPC)	Energy savings ranging from 17 % to 24 % were achieved.
Oldewurtel et al. [27], 2012	Model Predictive Control (MPC)	It was observed that air control directly predicted uncertain decisions in forecasting.
Ferreira et al. [28], 2012	Neural networks-based predictive control	Approximately 50 % energy savings were achieved.
Garnier et al. [29], 2014	Artificial Neural Network (ANN)-based model predictive controller	Compared to the existing strategy, up to 65 % energy savings were achieved.
Afram and Janabi-Sharifi [30], 2015	Gray-box modeling and control system design	The developed model was obtained with high accuracy, and it was observed to predict output variations precisely.
Huang et al. [31], 2015	A neural network-based multi-zone modeling approach for predictive control system design	More accurate prediction results were obtained compared to the single-zone model.
Barrett and Linder [32], 2015	Reinforcement learning approach	Energy loss prevention and thermal comfort were ensured.
Attaran et al. [33], 2016	PID control method with a new optimization algorithm based on epsilon constraint-RBF neural network	It was observed that thermal comfort was achieved while reducing energy consumption.
Wei et al. [34], 2017	Deep reinforcement learning (DRL)	Thermal comfort was ensured, and energy savings were achieved.
Homod [35], 2018	Fuzzy inference method with hybrid layered control algorithm	The studies showed that the system was conducive to energy savings.
Chen et al. [36], 2019	Gnu-RL, an advanced reinforcement learning-based control system	Gnu-RL provided 16.7 % savings in cooling demand compared to the existing controller.
Ding et al. [37], 2019	Deep reinforcement learning (DRL)	Compared to the latest DRL-based method in the literature, it achieved 8.1 % energy savings.
Gao et al. [38], 2020	Reinforcement learning (RL)	Experimental results showed that the method could improve thermal comfort prediction performance by 14.5 %, reduce HVAC energy consumption by 4.31 %, and enhance users' thermal comfort by 13.6 %.
Brandi et al. [39], 2020	Deep reinforcement learning (DRL)	Depending on the specified scenario, heating energy savings of approximately 5 %–12 % were achieved.
Kou et al. [40], 2021	Model- and data-based control method	It was concluded that combining the advantages of model-based and data-driven methods could provide better control performance and higher computational efficiency.
Du et al. [41], 2021	Deep deterministic policy gradient (DDPG)	When comparing the Deep Q Network (DQN) with the DDPG-based HVAC control method, it was shown that energy consumption costs could be reduced by 15 %, comfort violations by 79 %, and compared to a rule-based HVAC control strategy, comfort violations could be reduced by 98 %.
Deng et al. [42], 2022	Active change detection and deep reinforcement learning	Energy savings of 13 % and thermal comfort improvements of 9 % were achieved.
Esrafilian-Najafabadi and Haghighat [43], 2022	Machine learning (ML) techniques	It was also shown that machine learning evaluation metrics (e.g., MAE and accuracy) had a weak to moderate correlation with the overall performance score.
Abuhussain et al. [44], 2023	Fuzzy Controller Approach	Compared to fuzzy approaches, the proposed method showed better performance in maintaining the desired comfort level with lower energy consumption.

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Table 1 (*continued*)

References	Proposed control methods	Results/findings
Chojecki et al. [45], 2023	Fuzzy Controller	With the proposed FLC for air handling units (AHU), at least 27.4 % better integral control quality indicators (IAE, ISE, ITAE, ITSE) and smaller supply air temperature variations were achieved. Compared to an untuned PID, energy consumption was found to be 12.7 % lower.
Al Sayed et al. [46], 2024	Reinforcement learning (RL) tabanlı review	It was concluded that the high computational cost associated with the retraining of RL-based control methods for real-time applications should be addressed.
Nguyen et al. [47], 2024	Deep reinforcement learning (DRL)	The Phasic Policy Gradient (PPG)-based method showed a 2 %–14 % reduction in energy consumption, an improvement in indoor temperature comfort, and a 66 % faster convergence rate compared to traditional methods.
Park and Kim [48],2024	Supervised learning based iterative learning control	While maintaining temperature comfort in approximately 97–98 % of annual working days, HVAC has provided significant energy savings of 6.5–7.6 % in annual energy consumption.
Long et al., [49], 2025	Improved snake optimizer algorithm based fuzzy controller	The proposed method was reported to achieve zero temperature error, less than 1.5 % humidity error, and a 40.8 % reduction in annual energy consumption compared to other methods.
Yao et al., [50], 2025	Reinforcement learning and fuzzy reasoning based control	On average, 6.16 % higher energy cost savings and 15.15 % better thermal comfort were achieved compared to RL methods that optimize only HVAC supply air temperature.
Liu et al., [51], 2025	Hybrid model-based predictive control	In the application of the proposed method in building scenarios, mean absolute error (MAE) of 0.27 °C and root mean square error (RMSE) of 0.24 °C were obtained.

Şengirgin and Pulat examined the variation over time in reaching desired thermal values and modeling a single-zone heating-ventilation system. Using the on-off control method, they investigated the impact of return air and outdoor temperature on system control. The numerical simulation of the system controlled with the on-off method was performed in Matlab/Simulink. Return air enables energy to be used efficiently and economically. However, exceeding the required value affects the quality of indoor air [12]. In their study, Yiğit et al. [13] aimed to control the HVAC system using fuzzy logic and conventional on-off methods to lower the ambient temperature by 5 °C. The study examined the temperature variation of the air-conditioned environment over time and how the amount supplied to the environment cools the air. According to the results obtained, it was observed that fuzzy logic control was more advantageous than on-off control in terms of ensuring comfort conditions and energy savings. Özbek and Eker utilized a model predictive control method to maintain the controlled area within the HVAC system in a continuously optimal state despite varying thermal loads and system disruptions. [14]. Balance equations for the system were developed. The study aimed to predict the system's future behavior using mathematical modeling and implement model-based predictive control as the most suitable control method. Below is a table summarizing the studies provided in the literature. The table highlights these studies' primary contributions and advantages to the literature.

Consequently, Table 1 compares the studies in the literature to better understand and compare them.

Since these systems have a wide range of use, studies are ongoing in the literature by different scientists with various control methods, modeling and compilation articles [52–60]. The main objective of this study is to develop and evaluate an intelligent control strategy for a dual-zone HVAC system with variable airflow, using a PSO optimized Fuzzy LQR controller, and to compare its performance against classical LQR and PSO-LQR methods under varying environmental conditions.

Specifically, the main contributions of our study are summarized as follows.

a The first primary contribution is enabling the HVAC system with decision-making capability by determining the weights of the foot contact points of the membership functions in the Fuzzy LQR control method using the PSO algorithm. This approach facilitates the optimal tuning of the critically important Q and R parameters for system control, marking a novel application of this method to HVAC systems.
b The second significant contribution is the combined use and comparative analysis of LQR, PSO-based LQR, and PSO-based Fuzzy LQR control methods within the HVAC context, providing a comprehensive performance evaluation.
c Another important contribution is the use of the MSE performance index for a quantitative comparison of these control methods, which strengthens the robustness of our results.

Compared to existing studies, which generally apply traditional control methods without such integrated optimization and fuzzy logic combinations, our approach provides a novel framework that enhances the control performance of HVAC systems. Therefore, the authors consider this study to be theoretically original. The remainder of this paper is as follows: Section 2 presents the Mathematical Model of the HVAC System. Section 3 describes the Particle Swarm Optimization (PSO) algorithm and the design of the LQR, PSO-based LQR, and PSO-based Fuzzy LQR control methods. Section 4 reviews Simulation Results and Discussion. In this section, numerical and graphical results obtained as a result of the study are given. Also, a comparison table with the literature and each other and its interpretation are presented at the end of this chapter Finally, section 5 summarizes the conclusion. The results of the article are analyzed and interpreted in this section. Also, at the end of this section, suggestions on how to improve the method even better and

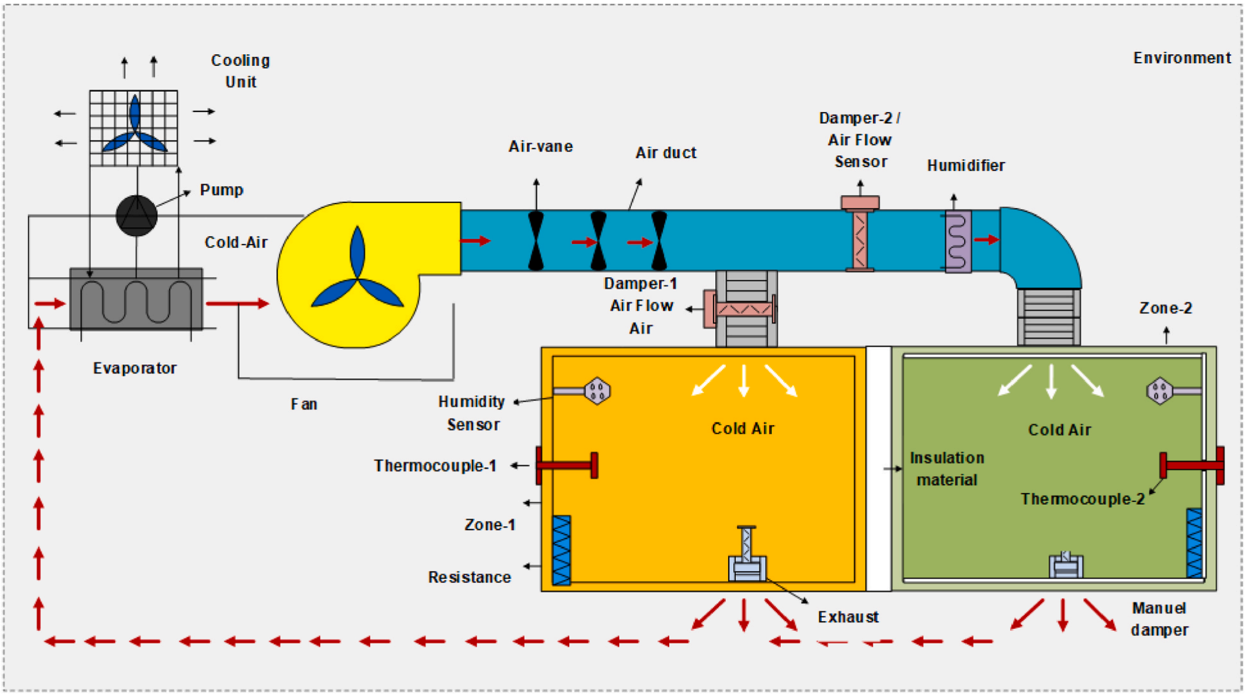


Fig. 1. Schematic representation of two-zone HVAC system.

information about future studies related to the method are given.

2. Mathematical model of the HVAC system

The control of HVAC systems in a simulation environment requires model equations. Modeling these systems is particularly challenging due to continuously varying temperature, humidity, and airflow. Therefore, certain assumptions were made while deriving the model equations. The assumptions include: the instantaneous changes in air velocity affect pressure, there is no air outlet in the zone other than the exhaust, airflow is homogeneous, and the amount of air entering and exiting the zone is equal. The system was modeled under these assumptions. The schematic representation of the two-zone HVAC system is shown in Fig. 1.

The various models of HVAC systems are used in the literature and this study is based on the model in Ref. [3] (Fig. 1). The HVAC system consists of cooled zone areas, an evaporator, a cooling unit, fans, ducts, damper motors, blades, thermocouples, humidity sensors, and resistors. Each zone area was designed with a volume of 0.5 m^3. The surfaces of Zone-2 were covered with insulation material (styrofoam), whereas this application was not implemented in Zone-1. The purpose of this type of design is to clearly see and determine the steady-state differences in order to obtain reference temperatures for two regions with different characteristics.

Cooled air is transferred to both zones from the main duct equipped with a supply fan. The temperature controls of both zones are performed by regulating damper opening ratios using the proposed controllers. The supply fan absorbs the 7 °C air obtained from the evaporator and then distributes it to both zones. The mass flow rate ($\dot{m}_{ca}$) of the air absorbed from the cooling unit remains constant because the supply fan operates at a fixed speed. However, the mass flow rates of the air entering the zones vary depending on the temperatures of the zones. The continuous variations in the mass flow rates of air entering the zones, $\dot{m}_{z1_{a,in,}}$ and $\dot{m}_{z2_{a,in}}$ are controlled by adjusting the opening ratios of dampers at the zone-duct inlets based on control output signals. The continuity equation of the HVAC system is constructed as given in Equation (1) below.

$$\dot{m}_{ca} = \dot{m}_{z1_{a,in}} + \dot{m}_{z2_{a,in}} + \dot{m}_{sva_{out}} \tag{1}$$

The mass flow rate given in the continuity equation (Equation (1)) $\dot{m}_{sva_{out}}$ belongs to the safety valve that discharges the excess air coming from the zones. The variable inlet mass airflows of the zones are provided by damper motors. Since it is assumed that there is no change in the flow rates of the zones from the inlet onward, the flow rate equals the outlet flow rate. Therefore, the continuity equation can be written as follows:

Based on the assumption that the amount of air entering and exiting the zone is equal, and thus there is no change in the amount of air, the continuity equation can be derived as follows (Equation (2)).

$$\dot{m}_{z_{a,in}} = \dot{m}_{exh_{out}} = \dot{m}_{z_a} \tag{2}$$

According to the first law of thermodynamics, the internal energy equation (Equation (3a)) can be expressed as:

$$Q–W + \sum U_{in} - \sum U_{out} = \frac{du}{dt} \tag{3a}$$

$$\sum U_{in} = \sum \dot{m}_{za_{in}}.\, h_{in} \tag{3b}$$

$$\sum U_{out} = \sum \dot{m}_{exh_{out}}.\, h_{out} \tag{3c}$$

$$Q - W + \sum \dot{m}_{za_{in}}.\, h_{in} - \sum \dot{m}_{exh_{out}}.\, h_{out} = \frac{du}{dt} \tag{3d}$$

here, u represents the time-dependent change of heat. In these equations, Q represents the heat of convection and conduction (*J*), W is the work (*J*), h denotes specific enthalpy [*J/kg*], and the subscripts '*in*' and '*out*' refer to inlet and outlet conditions, respectively. If it is assumed that no work is done in the system, Equation (3a) can be rewritten in the following form. The time-dependent change in internal energy:

$$\frac{du}{dt} = (m_{za}\, .C_v).\, \frac{dT}{dt} \tag{4a}$$

$$Q + \dot{m}_{za}\, .\, (\, h_{in} - h_{\,out}\,) = \frac{du}{dt} = \frac{(m_{za}\, .C_v).(T_{n-1} - T_n)}{dt} \tag{4b}$$

$$h_{in} = C_p.\, \left(T_{ca,in} \right) \tag{4c}$$

$$h_{out} = C_p.\, (\, T_n) \tag{4d}$$

$$h_{in} - h_{out} = C_p.\, \left(T_{ca,in} - T_n \right) \tag{4e}$$

In the equations given in 4a–4e, several variables are used to represent key thermodynamic properties. C_p (kJ/(kg·K)) denotes the specific heat at constant pressure, while C_v (kJ/(kg·K)) represents the specific heat at constant volume. $T_{ca,in}$ (°C) refers to the inlet temperature of the inner cooling air, and T_n (°C) indicates the instantaneous temperature at a given time. Finally, $\dot{m}_{za}$ (kg/s) is the mass flow rate of air entering Zone 1. When Equation (4b) is rearranged, Equations (5) and (6) are derived.

$$Q + \dot{m}_{za}\, .C_p.\, \left(T_{ca,in} - T_n \right) = \frac{m_{za}.C_v.\, (\, T_{\,n-1} - T_n)}{dt} \tag{5}$$

$$Q + \dot{m}_{za}.\, C_p.\, \left(T_{ca,in} - T_n \,\right) = m_{za}.\, C_v.\, \frac{du}{dt} \tag{6}$$

Here, *T* represents the instantaneous temperature change. The heat equation resulting from conduction and convection from the external environment into the system can be expressed as follows:

$$R = \frac{L}{A\, .\, k} \tag{7}$$

$$Q = \frac{T_{out} - T_n}{R} = \frac{T_{out} - T_n}{\frac{1}{h_{out}.A} + \frac{L_1}{k_1.A} + \frac{L_2}{k_2.A} + \frac{1}{h_{in}.A}} \tag{8}$$

The first-order differential heat equation of the zone model in its final state can be formulated as follows:

$$\frac{dT}{dt} = \frac{Q + \dot{m}_{za}.\, C_p.\, \left(T_{ca,in} - T_n \,\right)}{m_{za}.\, C_v} \tag{9}$$

In Equations (7)–(9), the parameter k (*J/m·K*) represents the thermal conductivity, also referred to as the transmission coefficient. The variable L (*m*) denotes the thickness of the respective zone or material layer through which heat is transferred, while A (m^2) corresponds to the cross-sectional area perpendicular to the direction of heat flow. The thermal resistance R (*K/W*) is calculated based on these parameters using the relation $R = \frac{L}{A\,.\,k}$.

3. Controller designs for the HVAC system

The HVAC system control aims to increase comfort through controllers while ensuring energy efficiency and designing a controller with a minimum error value. The purpose of these controllers, designed for the HVAC system, is to obtain the target temperature values of the two zones. The HVAC system was controlled using control methods such as Linear Quadratic Regulator (LQR), Particle Swarm Optimization-based Linear Quadratic Regulator (PSO-LQR), and PSO-based Fuzzy Linear Quadratic Regulator (PSO-FLQR). While designing the controller, the model equations of the HVAC system were used for system control in the simulation environment. The

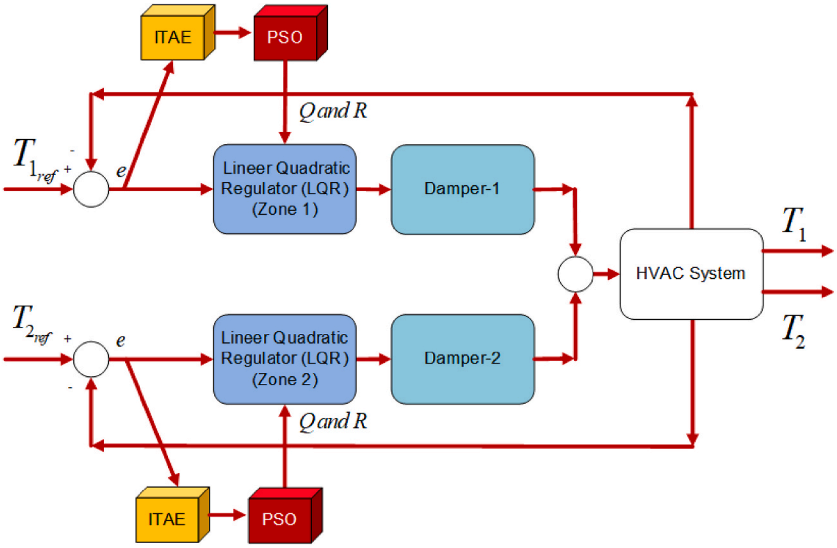


Fig. 2. Block diagram of PSO based LQR control method.

Linear Quadratic Regulator (LQR) control method, one of the optimal control methods using state-space models, is a type of controller based on the principle of full-state feedback [49]. The LQR control method is widely used because it is simple, optimal, and robust [61–63].

$u = -K * x$ the equation given shows the input of the LQR control method. Here, K denotes the feedback control input, and x represents the states of the system. While selecting a control input that minimizes the cost function, the state-space equations of the system are used (Equation (10)).

$$J = \frac{1}{2}\int_0^t \left(x^T(t)Qx + u^T Ru\right)dt \tag{10}$$

The aim of the control here is to minimize the integral of the quadratic performance index. Q and R are the weight matrices and Q is a positive semi-definite symmetric matrix and R a positive definite matrix ($Q \geq 0, R > 0$). In optimal control, u is the control vector, and x represents the states of the system. Equation (10) is quadratic with respect to both x(t) and u(t). In Equation (11), K optimal feedback input is given.

$$K = R^{-1}B^T P \tag{11}$$

B and P show the input matrix and positive definite matrix of the system, respectively, and also P matrix is obtained with the help of Riccati equation. Equation (12), which represents the Algebraic Riccati Equation (ARE), was solved numerically using MATLAB's built-in lqr function, which provides an optimal state feedback gain matrix by internally solving the ARE. In Equation (12), A denotes the state matrix.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{12}$$

Various studies are being carried out on how to obtain the Q and R matrices, which are of critical importance in the LQR control method, in an optimum manner. In this study, the Particle Swarm Optimization (PSO) algorithm was used to obtain the Q and R parameters of the LQR control method. The PSO algorithm, which is one of the swarm-based meta-heuristic optimization methods, was preferred [64,65]. The integral of the time-weighted absolute error (ITAE) of the LQR control method parameters was used as the objective function for PSO (Equation (13)). In the PSO algorithm, the number of particles is 30 and the number of iterations is 100.

$$G = ITAE_{T_{zone1}} + ITAE_{T_{zone2}} = \int_0^t t\left|e_{T_{zone1}}\right|dt + \int_0^t t\left|e_{T_{zone2}}\right|dt \tag{13}$$

$e_{T_{zone1}}$ zone-1, 1. represents the temperature control error of zone 1 and $e_{T_{zone2}}$ zone-2 represents the temperature control error of zone 2. The working time is shown in t The block diagram of the PSO-based LQR control method is shown in Fig. 2.

The fuzzy logic algorithm proposed by Zadeh is a method consisting of five stages. In the first stage, fuzzification involves converting input variables into a fuzzy set. The second stage consists of rule tables, membership functions, and a rule base [66]. This rule base is a group of IF-THEN rules derived from the verbal expressions of experts with knowledge about the system [67–70]. The third stage, the inference mechanism, executes the inference process of these rules on the system and provides fuzzy outputs. The fourth stage called the database, defines the membership functions and the universe of discourse for the fuzzy sets. The final stage, defuzzification, converts a fuzzy set into a crisp value for the output. Another proposed method for the control of HVAC systems is the Fuzzy

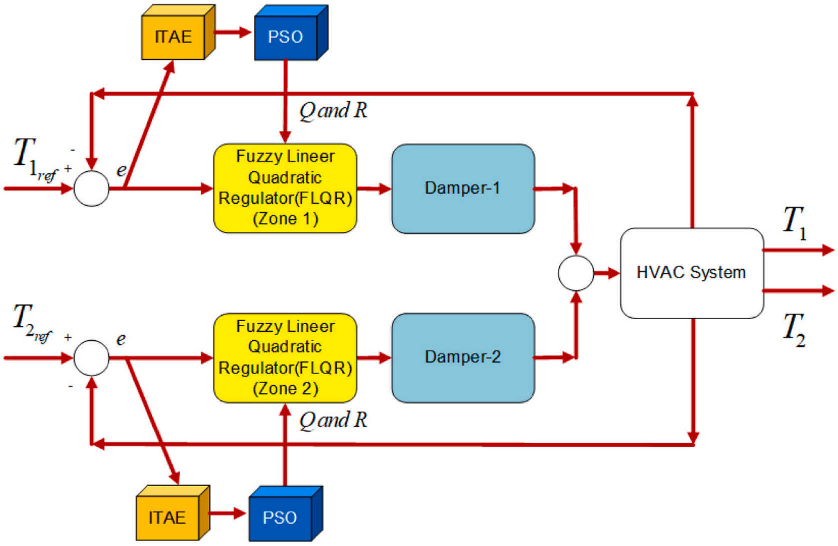


Fig. 3. Block diagram of PSO-based FLQR control method.

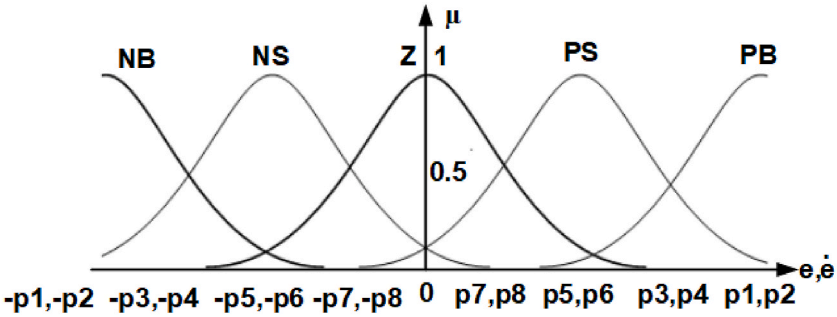


Fig. 4. PSO-FLQR membership functions defined for input values $\boldsymbol{e}$ and $\dot{\boldsymbol{e}}$.

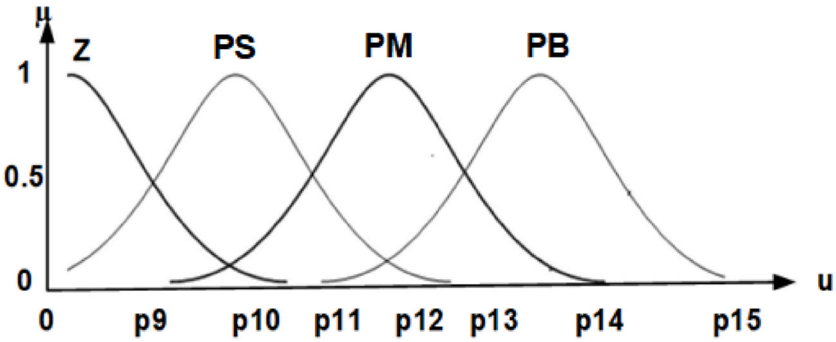


Fig. 5. PSO-FLQR membership functions defined for output value u

Linear Quadratic Regulator (FLQR) control method, which combines the advantageous features of the Linear Quadratic Regulator (LQR) and Fuzzy Logic control methods. Obtaining the Q and R matrices, which influence the performance of the LQR control method, using fuzzy logic transforms the proposed method (FLQR) into a dynamic structure based on variable temperature values.

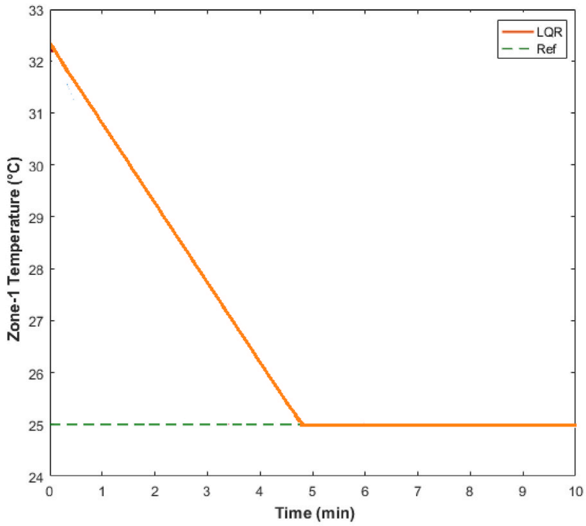
This aims to enhance the system's performance under variable conditions, allowing the method to exhibit a performance closer to reality. The FLQR controller, which uses the error ($\boldsymbol{e}$) and the derivative of the error ($\dot{\boldsymbol{e}}$) as inputs, employs fuzzy controller rules to modify the state feedback gain expressions. The boundary values of the membership functions for the FLQR control algorithm were obtained using the Particle Swarm Optimization (PSO) algorithm. The integral of time-weighted absolute error (ITAE) was determined as the objective function for PSO. In the PSO algorithm, the particle number was set to 30, and the iteration count was set to 100. Fig. 3

Table 2
The limit values of FLQR membership functions.

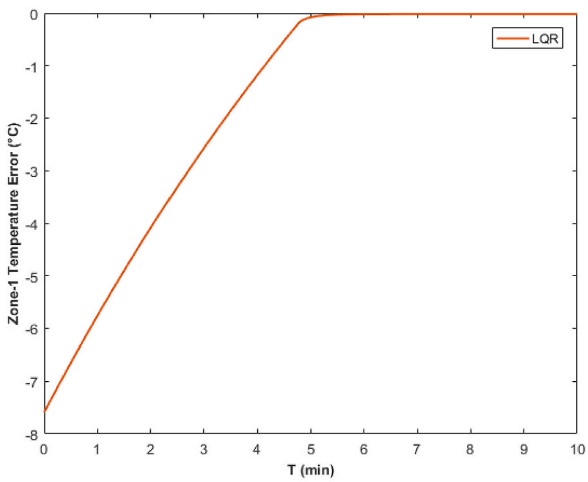
p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
1	0.8	0.71	0.64	0.47	0.39	0.24	0.18
p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	
0.31	0.42	0.57	0.63	0.72	0.84	1	

Table 3
The rule table created for FLQR.

u	NB	NS	Z	PS	PB
NB	*NB*	*NB*	*PS*	*PB*	*NS*
NS	*NS*	*PM*	*Z*	*NS*	*PB*
Z	*Z*	*NS*	*PM*	*PB*	*PM*
PS	*PM*	*NB*	*PB*	*PS*	*PB*
PB	*PB*	*PB*	*NB*	*PS*	*PB*

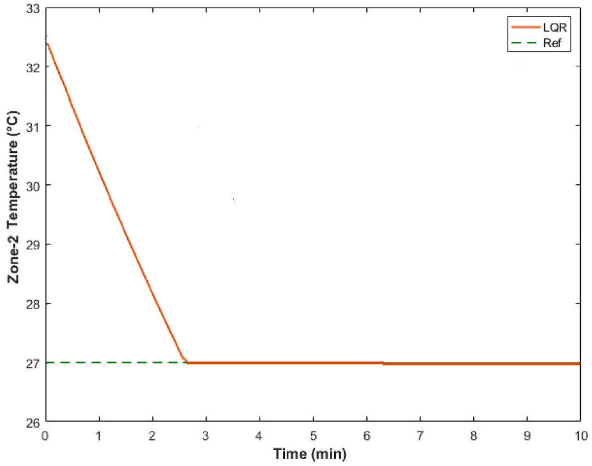


a)

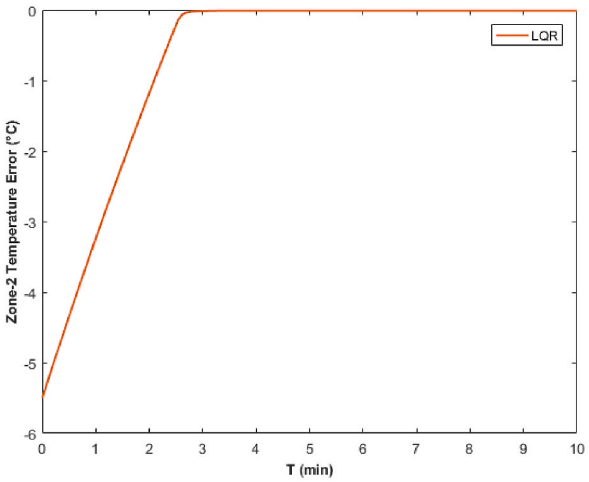


b)

Fig. 6. When LQR method is applied for Zone-1 **a)** Temperature change and **b)** error graph obtained.



a)



b)

Fig. 7. When LQR method is applied for Zone-2 region, **a)** Temperature change and **b)** error graph.

presents the block diagram of the PSO-based FLQR control method.

The controller uses the error ($\boldsymbol{e}$) and the rate of change of errors ($\dot{\boldsymbol{e}}$) as input values. In the PSO-FLQR control method, the Mamdani method and Gaussian-type membership functions were utilized. The membership functions and the rule base created for PSO-FLQR are provided below. Fig. 4 shows the membership functions and boundary values defined for the system's input values e and $\dot{e}$. The membership functions and limit values defined for the output values (temperature values) are shown in Fig. 5.

Table 2 provides the boundary values of the membership functions used in the PSO-FLQR control method, optimized using the PSO algorithm. Table 3 presents the rule table created for the PSO-FLQR control method. In the table, the fuzzy control variables $e, \dot{e}, u$ represent error, error change, and u, respectively. The abbreviations NB, NS, Z, PS, PM, and PB stand for Negative Big, Negative Small, Zero, Positive Small, Positive Medium, and Positive Big, respectively.

4. Simulation Results and Discussion

In this section, the simulation results obtained by testing the controllers designed for an HVAC system on the system are presented. In this study, the control of a two-zone HVAC system was conducted. The results obtained using the model equations and control methods of the HVAC system will be presented in this part. The HVAC system was controlled using LQR, PSO-LQR, and PSO-FLQR control methods. The results obtained will be provided numerically and graphically. The control of the two-zone HVAC system, with each zone having a different temperature value, was performed. Based on meteorological data, the ambient temperature value was taken as 32.2 °C, the highest average temperature in Muş city center during the first week of August (August 1–7). In this two-zone region, the evaporator delivers air taken at 5 °C with a constant flow rate to the zones with a variable flow rate. The dampers located at the zone inlets adjust the airflow entering the environment by varying between 0° and 90° depending on temperature changes. It is

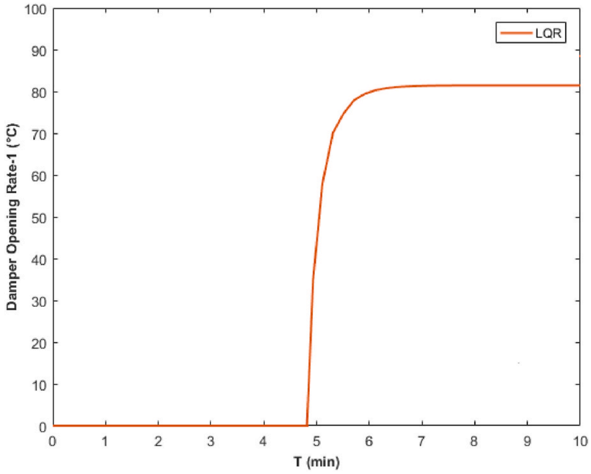


Fig. 8. Damper opening change obtained when LQR control method is applied for Zone-1.

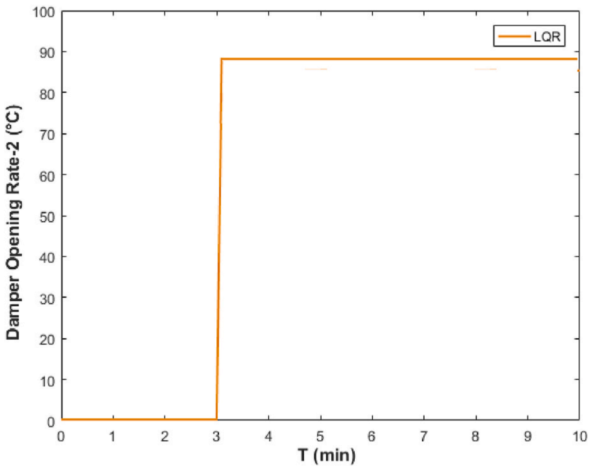
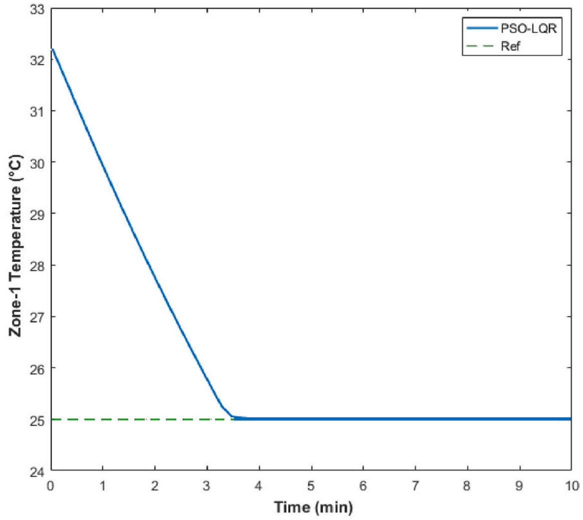


Fig. 9. Damper opening change obtained when LQR control method is applied for Zone-2.

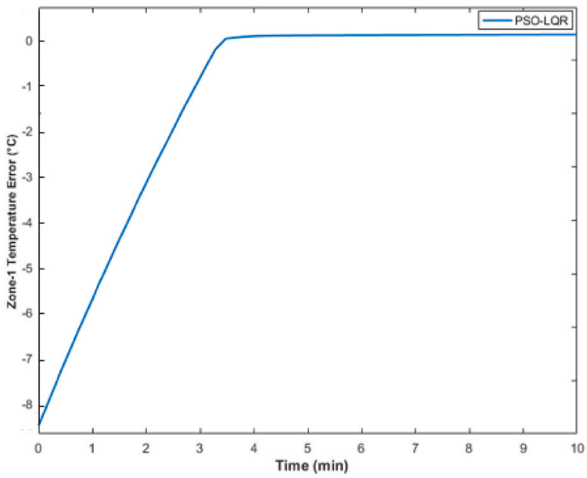
assumed that at the starting position of the damper, at 90°, there is no air entry, while at 0°, maximum air entry is allowed. The ambient temperature for Zone 1 and Zone 2 regions was taken as 32.2 °C. The reference indoor temperatures for Zone 1 and Zone 2 were set to 25 °C and 27 °C, respectively, and these regions were cooled to the specified temperature values. The channel section ratio of the dampers affecting the damper opening rate was taken as 0.05 m^2. The simulation duration was set to 10 min. The results obtained with the LQR control method, one of the control methods designed and applied for the HVAC system, are presented in this section. The temperature variation and temperature error graph for Zone 1, obtained as a result of the method's application, are provided in Fig. 6a and b. Similarly, the temperature variation and temperature error graph for Zone 2 are provided in Fig. 7a and b.

As a result of applying the LQR control method to the system, the desired temperature value from the ambient temperature was reached in approximately 5.5 min (Fig. 6a and b). Additionally, it was observed that Zone 1 stabilized with an error of approximately 5.5 min.

As a result of applying the LQR control method to the system, Zone 2 reached the desired temperature value in approximately 3 min (Fig. 7a and b). Additionally, it was observed that Zone 2 stabilized with an error of approximately 3 min. The damper opening rate variation graphs for Zone 1 and Zone 2 obtained from the LQR control method applied to the HVAC system are provided in Figs. 8 and 9.



a)

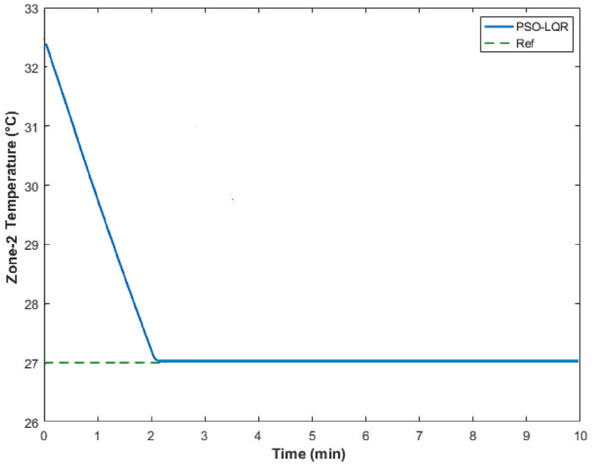


b)

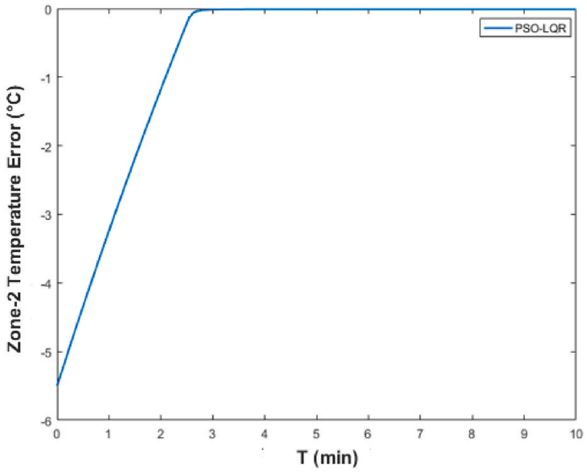
Fig. 10. When PSO- LQR method is applied for Zone-1 region, **a)** Temperature change and **b)** error graph.

As seen in Fig. 8, the damper belonging to Zone 1 starts to close after 4.8 min when the ambient temperature of Zone 1 reaches the reference temperature value.

The damper belonging to Zone 2 starts to close approximately 3 min after the ambient temperature of Zone 2 reaches the reference temperature value (Fig. 9). The results obtained using the PSO-based LQR control method, another control method designed and applied for the HVAC system, are presented in this section. The temperature variation and temperature error graph for Zone 1, obtained as a result of the application of the PSO-LQR control method to the HVAC system, are provided in Fig. 10a and b. Similarly, the temperature variation and temperature error graph for Zone 2 are provided in Fig. 11a and b.



a)



b)

Fig. 11. When PSO- LQR method is applied for Zone-2 region, **a)** Temperature change and **b)** error graph.

As a result of applying the PSO-LQR method to the system, it was observed that the desired temperature value was reached in approximately 4 min, which is a shorter duration compared to the LQR control methods (as shown in Fig. 10a and b). Additionally, Zone 1 achieved a settling time of approximately 4 min.

When the PSO-LQR control method was applied to the system, the desired temperature value for Zone 2 was reached approximately 2.6 min from the ambient temperature (as shown in Fig. 11a and b). Furthermore, Zone 2 stabilized with an error of approximately 2.6 min. The damper opening rate variation graphs for Zone 1 and Zone 2, obtained as a result of the PSO-LQR control method applied to the HVAC system, are provided in Figs. 12 and 13.

As shown in Fig. 12, the damper belonging to Zone 1 starts to close after 4 min, when the ambient temperature of Zone 1 reaches the

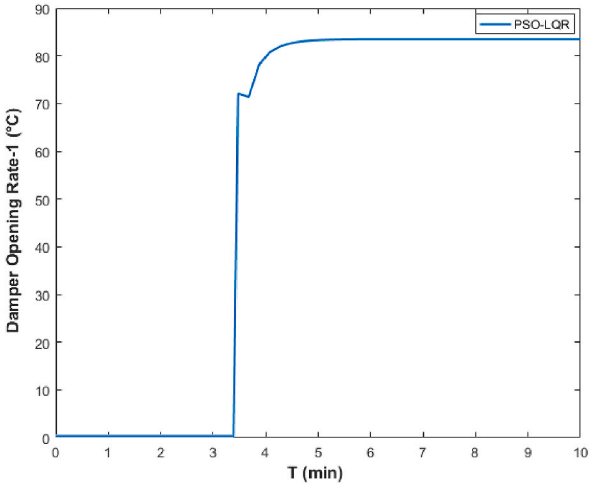


Fig. 12. Damper opening change obtained when PSO-LQR method is applied for Zone-1.

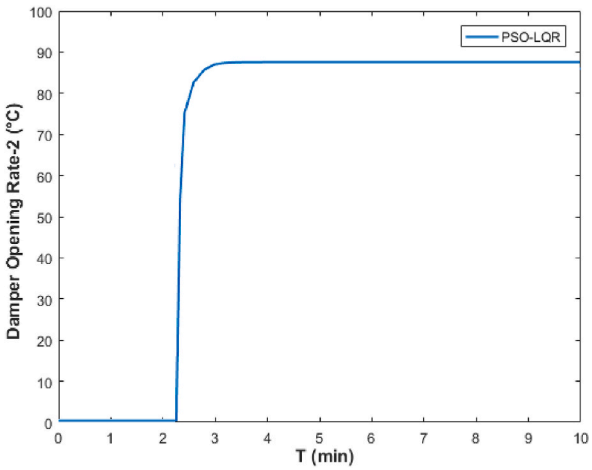


Fig. 13. Damper opening change obtained when PSO-LQR method is applied for Zone-2.

reference temperature value.

The damper belonging to Zone 2 begins to close after approximately 2.6 min when the ambient temperature of Zone 2 reaches the reference temperature value (as shown in Fig. 13). The results obtained using the PSO-FLQR control method, another control method applied to the HVAC system, are presented in this section. The temperature variation and temperature error graph for Zone 1, obtained as a result of applying the PSO-FLQR control method to the HVAC system, are shown in Fig. 14a and b. Similarly, the Zone 2 temperature change and temperature error graph is given in Fig. 15a and b.

As a result of applying the PSO-FLQR method to the system, the desired temperature value was reached in approximately 2 min, a shorter time compared to the LQR and PSO-LQR control methods. Additionally, it was observed that stabilization occurred with minimal error (as shown in Fig. 14a and b). Moreover, Zone 1 had a settling time of approximately 2 min.

When the PSO-FLQR method was applied to the system, the desired temperature value for Zone 2 was reached in approximately

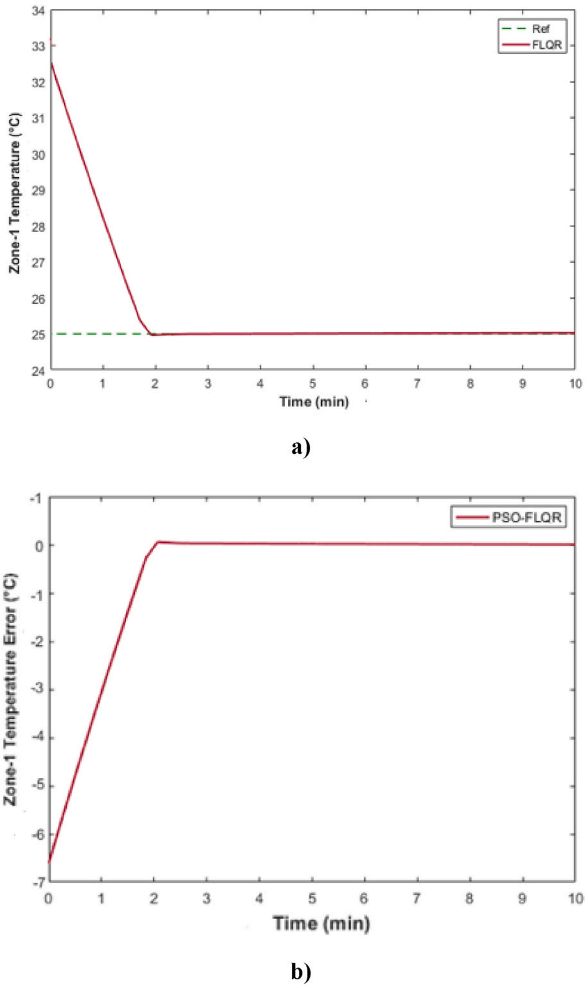


Fig. 14. When PSO-FLQR method is applied for Zone-1 region **a)** Temperature change and **b)** error graph obtained.

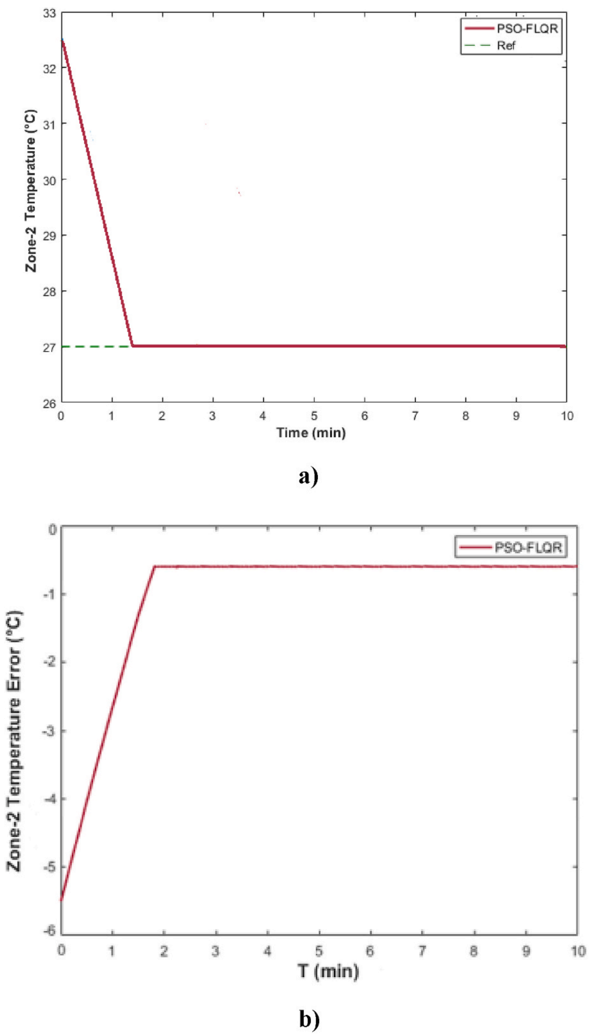


Fig. 15. When PSO-FLQR method is applied for Zone-2 region a) Temperature change and **b)** error graph obtained.

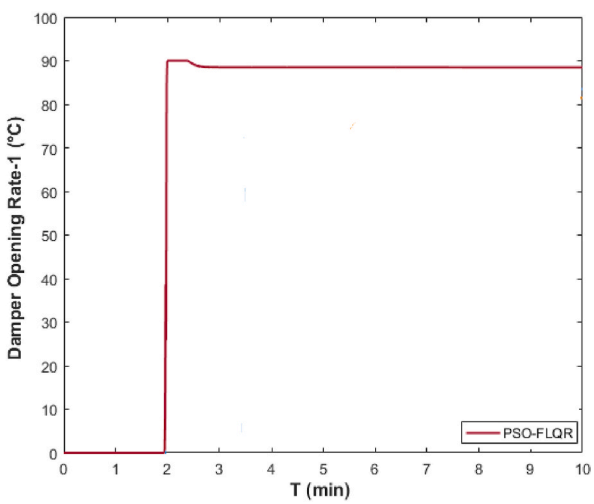


Fig. 16. Damper opening change obtained when PSO-FLQR method is applied for Zone-1 region.

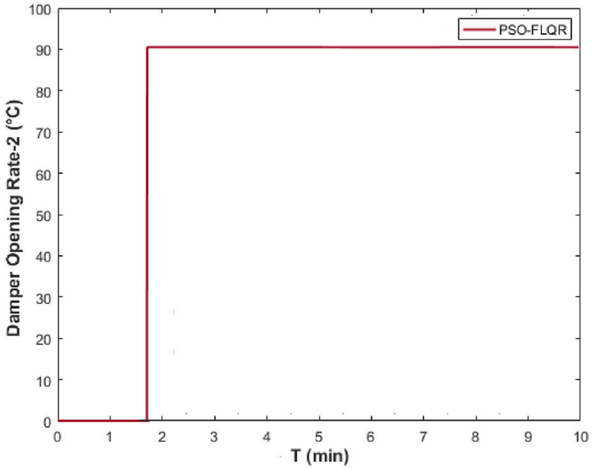


Fig. 17. Damper opening change obtained when PSO-FLQR method is applied for Zone-2 region.

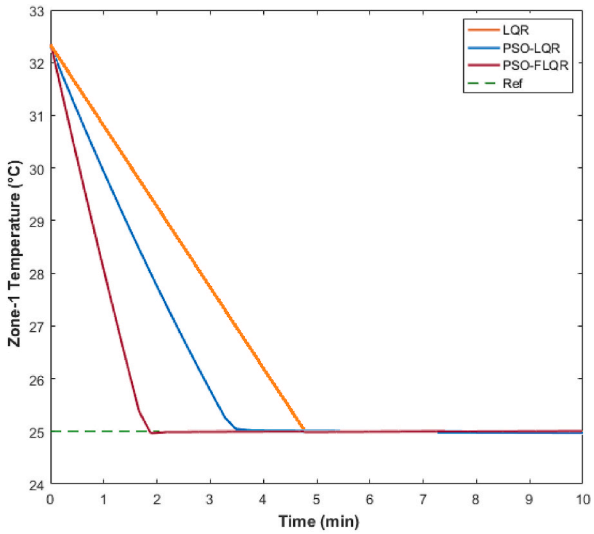


Fig. 18. Comparison graph of damper opening change of control methods applied for Zone-1 region.

1.8 min, a shorter duration compared to the LQR and PSO-LQR control methods. Furthermore, it was observed that Zone 2 stabilized with minimal error (as shown in Fig. 15a and b). Zone 2 also had a settling time of approximately 1.8 min. Compared to other methods, PSO-FLQR demonstrated the best performance, achieving the shortest settling time and no steady-state error for both Zone 1 and Zone 2. The damper opening rate variation graphs for Zone 1 and Zone 2, obtained as a result of the PSO-FLQR control method applied to the HVAC system, are shown in Figs. 16 and 17.

As observed in Fig. 16, the damper belonging to Zone 1 starts to close after 2 min, when the reference temperature value for Zone 1 is reached.

The damper belonging to Zone 2 begins to close after approximately 1.8 min when the reference temperature value for Zone 2 is reached (as shown in Fig. 17). The comparative temperature variation graphs for Zone 1 and Zone 2, based on the control methods applied to the HVAC system, are shown in Figs. 18 and 19.

The performance of the PSO-FLQR control method applied to both zones demonstrated the best results among the LQR and PSO-LQR control methods in terms of both steady-state error and settling time (Figs. 18 and 19). There was no steady-state error, and the

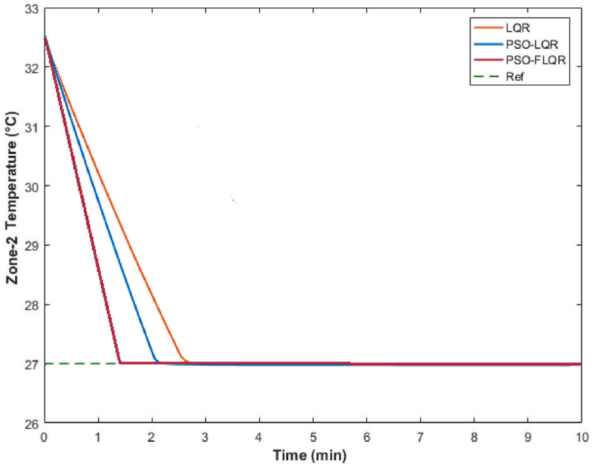


Fig. 19. Temperature change comparison graph of the control methods applied for Zone-2 region.

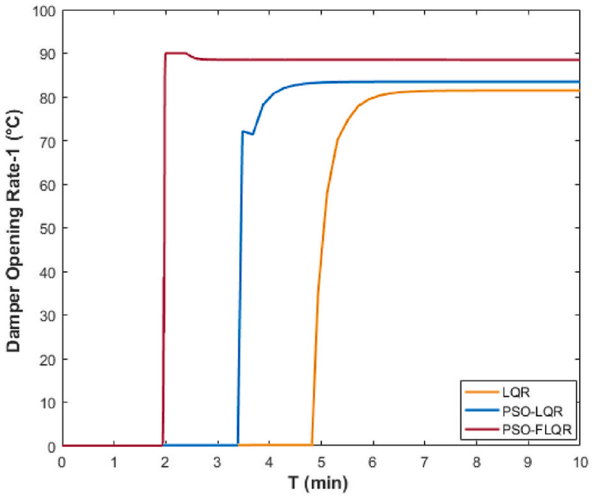


Fig. 20. Comparison graph of damper opening change of control methods applied for Zone-1 region.

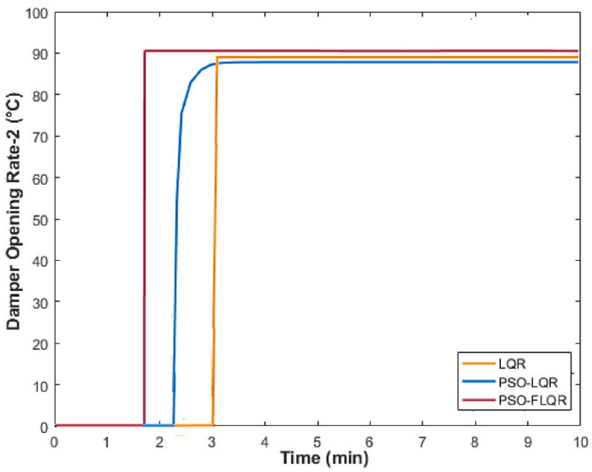


Fig. 21. Comparison graph of damper opening change of control methods applied for Zone-2 region.

Table 4
Zone-1 and Zone-2 temperature error values (°C).

Control Methods/MSE	Zone-1	Zone-2
LQR	0.983	0.412
PSO-LQR	0.217	0.126
PSO-FLQR	**0.094**	**0.049**

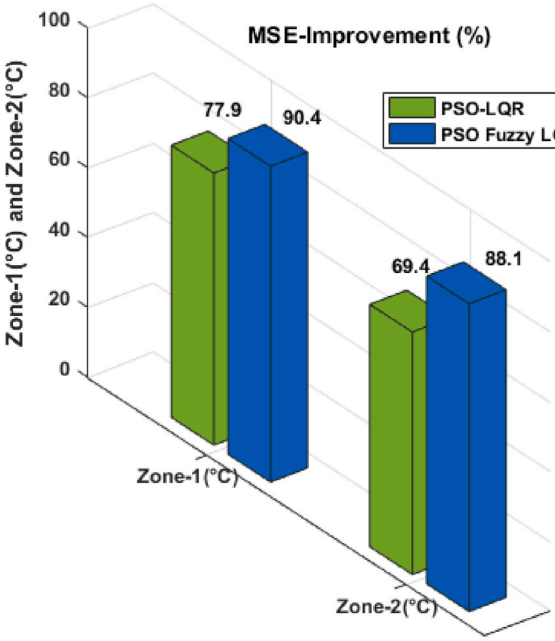


Fig. 22. The percentage performance comparisons of the zone temperatures.

Table 5
Comparison of the PSO based Fuzzy-LQR control method with existing methods.

Performance Criteria (ITAE)	Zone-1	Zone-2
PID	1.542	1.317
LQR	1.237	0.904
PSO-LQR	0.384	0.269
PSO-FLQR	**0.142**	**0.103**

system reached the desired reference temperature while minimizing the settling time. Although the PSO-LQR controller also exhibited no steady-state error, its settling time was longer. The graphs comparing the damper opening ratio changes for Zone-1 and Zone-2 under the control methods are presented in Figs. 20 and 21.

As observed in Figs. 20 and 21, the PSO-FLQR control method showed the best performance in terms of the damper opening ratio changes, starting immediately after each zone reached its reference temperature value. A comparison of the temperature error values for the control methods applied to Zone-1 and Zone-2 based on performance criteria is presented in Table 4. The values in Table 4 were obtained using the Mean Square Error (MSE) performance criterion (Equation 14a-b).

$$MSE_{T_{zone1}} = \int_0^t \sum \left(e_{T_{zone1}}{}^2\right) \tag{14a}$$

$$MSE_{T_{zone2}} = \int_0^t \sum \left(e_{T_{zone2}}{}^2\right) \tag{14b}$$

The performance error values of the LQR, PSO-LQR, and PSO-FLQR control methods given in Table 4 are highlighted in bold to indicate the lowest error values clearly. The highest temperature error values for both zones across all methods were obtained with the LQR method. The lowest temperature error values were obtained using the PSO-FLQR control method. Analyzing all error results obtained using the MSE criterion shows that the proposed PSO-FLQR control method demonstrated superior performance compared to other methods applied to the system. The percentage improvement in performance over the LQR control method for the applied methods was calculated using Equation (15). The percentage improvement graphs of the methods compared to the passive system are shown in Fig. 22.

$$Improvement(\%) = \left| \frac{first\ method - second\ method}{first\ method} \right| * 100 \tag{15}$$

According to the graphs in Fig. 22, using the PSO-based Fuzzy LQR control method increased control performance by 90.4 % for Zone-1 and 88.1 % for Zone-2 compared to the LQR method. As seen in Fig. 22, among the three control methods applied, the PSO-based Fuzzy LQR method achieved maximum performance improvement percentages across all parameters.

To demonstrate the performance of the proposed PSO-based FLQR control method, the Integral Time Absolute Error (ITAE) criterion has been employed. The results are compared with those of the conventional LQR, PSO-based LQR, and classical PID control methods. The comparative values are summarized in Table 5.

The performance error values of the LQR, PSO-LQR, PSO-FLQR, and PID control methods presented in Table 5 are highlighted in bold to clearly emphasize the lowest error values. The analysis of all error metrics based on the ITAE criterion demonstrates that the proposed PSO-FLQR control method outperforms the other control strategies applied to the system. The limitations and practical applicability of the method (PSO-FLQR) proposed in this work can be stated as follows. Although the PSO-based FLQR method has shown superior performance in terms of minimizing temperature error and improving comfort levels, it requires more computational resources compared to the standard LQR or PSO-LQR methods. However, it is important to note that the PSO optimization is performed offline before system deployment. Once the optimized fuzzy rule base and weight matrices are determined, the online execution primarily involves fuzzy inference and state feedback computation, both of which can be efficiently processed by modern embedded processors or digital controllers. Therefore, the PSO-FLQR controller remains applicable for real-time implementation, especially in systems with sufficient computational capacity.

5. Conclusions

In this study, the modeling and control of a two-zone HVAC system were carried out using Linear Quadratic Regulator (LQR), Particle Swarm Optimization (PSO)-based LQR, and PSO-based Fuzzy LQR (FLQR) intelligent control methods. Based on simulation results, including graphical and MSE performance criteria (see Table 4), the PSO-FLQR method achieved the lowest temperature errors of 0.094 °C and 0.049 °C for Zone-1 and Zone-2, respectively, demonstrating superior control accuracy compared to other methods. The study quantifies the performance improvement of the PSO-FLQR method as 90.4 % for Zone-1 and 88.1 % for Zone-2 relative to the classical LQR control, confirming its effectiveness in maintaining comfort conditions under varying environmental factors. Future research directions include implementing the proposed control strategies in real-time HVAC systems to validate their practical feasibility. Further, investigating other optimization algorithms or objective functions may enhance control performance. Additionally, extending the model to multi-zone systems with more complex environmental interactions and testing under diverse climatic conditions can provide deeper insights and applicability.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Declaration of competing interest

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Data availability

The authors do not have permission to share data.

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