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The Preservice Teachers' Approaches Toward Incorrect Probability Problems

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Abstract

Identifying the mistakes in student responses and being able to manage the solution process are among the fundamental skills that should be acquired by the pre-service teachers. This skill is important in probability, about which students have a great number of misconceptions. The aim of this study was to determine the misconceptions in the student answers given by the researcher about probability and to examine the solution explanations of the preservice teachers. In this context, the student answers containing five wrong solutions were examined. Case study, one of the qualitative research methods, was used. Forty-two third grade preservice mathematics teachers participated in the study. A task-based interview was used in the study. The findings revealed that the participants were able to recognize simple and discrete probability problems more easily and provided correct solutions; however, they had difficulty in determining the universal space independent events and failed to detect the mistakes in the student answers successfully. As a result of the research, it was concluded that the importance attached to procedural knowledge should also be given to conceptual knowledge, what a concept means as well as what it does not mean should be explained, open-ended questions that will increase the power of reasoning should be used instead of typical problems, and more effective learning should be carried out by using different teaching methods and techniques.

Keywords

probability, problem, preservice teachers, elementary school mathematics, misconceptions

Introduction

The basis of probability teaching, one of the topics in mathematics, is based on conceptual knowledge, in other words, the ability to make inferences and comment on events, rather than memorizing the formula or question solutions (Hawkins, 1990). Probability is the science that can examine Mathematics in its finest detail and deals with random or imprecise events (Arı, 2010). Probability emerged as a result of the multiplicity and complexity of the reasons for the existence of randomly occurring events and the needs of humanity to try events or situations (Ersoy & Erbaş-Oral, 1996). Statistics and probability is a science that facilitates an individual's daily life (Özmantar et al., 2008), develops independent and creative thinking skills based on mathematics, problem-solving skills based on reasoning, mental operations based on predictions and inferences, building models on questions, and verbal-mathematical expression of models (Arı, 2010).

The notion of probability, used in several areas of daily life such as lottery, risk analysis, insurance, is also used in different fields such as meteorology, quantum physics, and genetics. Probability, which is used from the weather forecast to support the proof of a mathematical result, can enable individuals to make the correct decisions in their life (Kazak, 2014). It exists in numerous areas both in daily life and different academic fields and is a significant part of the curriculum in many countries.

Probability is a considered as difficult by both teachers and students (Arı, 2010; Bozkurt & Akalın, 2010; Gürbüz, 2008; Koparan, 2015; Koparan & Kaleli Yılmaz, 2015; Memnun, 2008). Despite the fact that it is a mathematical field having a very old history and quite noteworthy applications in real life (economy, physics, medicine, and economics; Jones, 2005). Probability has been taught in schools as a mathematics topic for only a few decades (Franklin et al., 2007). Probability is a new

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topic which has a different reasoning structure and logical reasoning compared to other mathematical topics (Batanero et al., 2004; Hannigan et al., 2013). This is the most important reason for difficulties students and teachers experience both in teaching and learning.

The misconception occurs as a result of the fact that the concept conveyed by the teacher is not fully formulated by the student or due to previous mislearning. (Ausubel, 2000). Misconceptions are quite significant in mathematics since they lead to constant errors in students (Waluyo et al., 2019). In order to eliminate such problems, it is necessary to focus primarily on conceptual teaching. Then, the sources of these errors should be investigated, and solutions should be proposed (Akkaya, 2010; Öksüz, 2010). According to Fisher and Lipson (1986), students can correct their mistakes in three steps. The first step starts when the students realize the mistake. In the second step, the mistake is analyzed, and its reason is tried to be found. In the final step, a strategy to eliminate and correct the mistake is developed. So experienced and solved uncertainties mean mistakes which are revealed and analyzed by the pre-service teacher. The most significant contribution of mathematical mistakes is that they can lead to cognitive conflict in students. The student thinks and reflects on their thoughts to eliminate this cognitive conflict and thus, it becomes easier to learn the topic (Borasi, 1994). Soylu and Soylu (2005) argue that it is better for students not to study the topic and not to know anything, rather than to have misconceptions as a result of studying on their own. Therefore, the topics that the student will have difficulty or lead to misconceptions should be identified before the lesson and instructional activity should be carried out accordingly.

Mistake is defined by Erbaş et al. (2009) as misinterpretation and misuse of mathematical symbols and ideas. Borasi (1986, 1988, 1989, 1994) emphasized that it is important to think about the teaching of mistakes, that content knowledge will develop in this way, and that different situations may arise while thinking about the source of the mistake. In addition, it can be argued that mistakes play a role in the emergence of negative knowledge (Dalehefte et al., 2012).

Mistakes and uncertainties are significant especially in teaching probability since it is regarded as more uncertain and difficult due to its nature compared to other mathematical topics. The most important reason for this difficulty and uncertainty is that it has a different reasoning frame than other mathematical topics (Batanero et al., 2004; Hannigan et al., 2013). Its uncertain structure increases students' possibility to make mistakes in probability. Therefore, what preservice teachers need to know should be identified in Probability education.

There are several models for the knowledge preservice teachers are expected to have in probability education. In this sense, Pino-Fan et al. (2015) proposed an effective model for the skills that teachers are expected to have for probability education. In this model, they divided probability knowledge into two as common probability knowledge and extended probability knowledge. Common probability knowledge can be defined as the knowledge that is expected to be acquired by students from 8th to 12th grade in the education program of Turkey. In sum, preservice teachers should have the knowledge and concepts that are expected to be acquired by 12th-grade students (Ruz et al., 2021). This knowledge consists of the most basic information and concepts related to probability.

In addition, improving pre-service teachers' ability to evaluate students' mathematical knowledge will help them adopt an appropriate method for designing their lessons in the future. (Yao et al., 2021). In-depth examination of errors in probability during teacher education may enable preservice teachers to learn concepts related to probability.

The Purpose and Significance of the Study

The aim of the study was to investigate preservice mathematics teachers' error detection in student answers and the ways they dealt with these errors. This study focused on preservice teachers' common probability knowledge and their ability to produce a solution.

People often use probability in daily life while making inferences and reasoning in situations where they face uncertainty. The fact that the possibility of the occurrence of uncertain events depends on probability, prioritizes detail, attention, critical perspective, and intuitive thinking skill in understanding probability. In addition, logical estimations are based on a strong mathematical language, operation, and conceptual knowledge (Erdem, 2011). Determining what needs to be known about probability (Bradshaw et al., 2014) and developing new methods to improve knowledge are quite important as there should not be incomplete or incorrect knowledge about the subject for effective education (Ball et al., 2008; Godino et al., 2017; Shulman, 1986). Otherwise, effective education will not be provided (Batanero, 2013; Koparan, 2019; A. M. Leavy, 2010; Stohl, 2005). Preservice teachers who are not well-educated in probability may not feel comfortable teaching probability in the future or may not realize the importance of the topic (Batanero & Díaz, 2012). An adequate understanding of probability and its importance will contribute to a better understanding and making sense of individuals' life (Erdem, 2011).

According to Konyalıoğlu et al. (2010), the detection of error is closely related to learning since, using learned knowledge, an individual should be able to explain both correct and incorrect knowledge with its reasons. In this regard, detecting student mistakes improves one's content knowledge (L. Ma, 2010). Therefore, detecting existing mistakes in probability and the ways of dealing with them will improve both the content knowledge and the ability to think correctly. As problem posing and solving activities increase problem-solving skills (A. Leavy & Hourigan, 2020), evaluating problems will also improve pre-service teachers' problem-solving strategies. Thus, examining different solution strategies can improve pre-service teachers' problem-solving skills (Erdem et al., 2018).

The ability to detect student mistakes, which is also emphasized by National Council of Teachers of Mathematic (NCTM, 1989, 1991, 2000), requires teacher training institutions to address this skill. It is very important for preservice teachers to be able to notice the misconceptions and students' incorrect intuitions and to develop methods to correct these mistakes (Batanero & Borovcnik, 2016). It is known that the examination of student mistakes is an important tool that can be used to facilitate student development. Although there are several studies on error detection on probability in the literature, only one study focused on the ways of dealing with student errors (Demirci et al., 2017).

Teaching the subject of probability in a meaningful way depends on knowing the mistakes and misconceptions that students often make. It is also important for pre-service teachers to be able to identify student mistakes and develop a solution process for them, as well as their answers to a question. Because, although the correct answers from the pre-service teachers make them think that they have sufficient knowledge about the subject, being able to analyze the answers given by the students is for different skills. It is thought that teacher training institutions ignore this situation. So, while giving information about the subject to preservice mathematics teachers, they should also be informed about the mistakes that are usually made. In this direction, it is seen that the frequently made wrong solutions must be examined to the preservice mathematics teachers. Filling this gap in the literature, this study is significant in that it examines both preservice mathematics teachers' detection of student mistakes in probability and the ways they deal with them.

Method

Case study, one of the qualitative research methods, was adopted in the present study. Qualitative research, the aim of which is to obtain in-depth information about the phenomena being investigated (Merriam & Tisdell, 2016), helps to understand people's perspectives on life, their ways of structuring events, and their experiences (Merriam, 2013). In addition, case study examines what, how, and why of any event or situation that a limited sample works on (Yin, 2013). In this sense, the case study is effectively used in the field of education. It is important to use a unique case in a case study (Creswell, 2013). The study employed a qualitative research approach to thoroughly investigate the viewpoints of pre-service teachers regarding a problem they faced, their approach to formulating the correct solution, and how they applied their prior knowledge in this procedure. Furthermore, the case study was selected to scrutinize extensively the approaches adopted by the pre-service teachers, who were identified as the study participants, when confronted with incorrect solutions, their rationale for identifying such solutions as incorrect, and how they justified their correct solutions. In this study, the case of pre-service teachers' evaluation of probabilistic questions and solutions. They were asked to evaluate five probabilistic questions with solutions.

Participants

Forty-two preservice mathematics teachers in a state university in Eastern Anatolia of Turkey in the spring semester of the 2020 to 2021 academic year participated in the study. Criterion sampling, one of the purposive sampling methods, was used in sample selection. The criterion sampling, which is made up of people, events, objects, or situations with certain qualifications (Büyüköztürk et al., 2018), is based on the researcher's creation or bringing the situations that meet certain criteria together (Yıldırım & Şimşek, 2008). In this sense, the sample consisted of third grade pre-service teachers attending the teaching probability and statistics course. The purposeful sampling method was used to ensure that the participants had taken this course for 1 semester. In addition, the aim of using criterion sampling was to ensure that the pre-service teachers had learned the basic concepts of probability, probability types, probability simulations and probability distributions.

Data Collection Tool and Procedure

A task-based interview data collection tool, including five probability questions and their answers, was developed by the researchers in order to examine the participants' views on probability in depth. The task-based interview is a type of clinical interview in which the participant only interacts with the practitioner and with a task designed for a specific purpose (Goldin, 2000). In this study, the task consisted of five questions and answers. The questions were prepared by reviewing the literature and taking into consideration the questions that students most misunderstood. Based on the literature, two researchers in Mathematics Education framed the questions on probability and their incorrect solutions. Then, these questions were examined by both researchers, and the most appropriate five questions were selected. Then, the questions were examined by two different expert researchers. The attention was paid to include the misconceptions and errors about probability reported in the literature.

The study was carried out one-on-one with the participants. During the systematic data collection process, a series of clinical interviews were conducted with 42 pre-service teachers. These interviews took place twice a day, for a total of 10 sessions per week, and were conducted over a span of 31 days. Each pre-service teacher was allocated 30 minutes to evaluate the data collection tool. They were instructed to vocalize their thoughts and provide feedback on the given questions. Throughout the process, the researcher maintained silence, only intervening to clarify any aspects that the pre-service teacher found confusing. Furthermore, the researcher provided explanations for any verbal expressions that were unclear to the pre-service teacher. They were first asked to evaluate the solutions to the probability questions. They were also asked to express their thoughts verbally and in writing while evaluating the answers. The data were collected through video camera and written documents. Then, the data were transcribed. The data were categorized as correct solution, incorrect solution and no answer depending on correct detection and incorrect detection, and content analysis was performed. The aim of this study was not to investigate the effectiveness of a method for preservice teachers to learn probability. Instead, the qualitative research method was used in the study as preservice teachers' examination of misconceptions and errors is a very complex process and therefore it needs to be examined in depth.

The questions in the data collection tool are presented in Supplemental Appendix 1. Consistent with the ethical rules, the names of the participants were anonymized and codes such as S1, S2, and S42 were assigned to all participants. The analyzed data were interpreted and presented in tables. The codes determined by the researchers were classified along with the related categories and the findings were supported by direct quotations. In case the answers given are similar, the most original answer was preferred and different participants were presented with different examples for each question. The categories used in the study are as follows.

Incorrect Detection. Participants fail to identify the mistake or regard the mistake as correct.

Correct Detection. Participants successfully identify the mistake or express that student's answer is wrong.

Correct Solution. Participants provide a correct solution to the wrong answer.

Incorrect Solution. Participants provide a wrong solution to the wrong answer given or regard the mistake as correct and explain it.

Explanations of the categories and codes used in data analysis are presented in Table 1.

The first letter of the codes in Table 1 is *detection method*; the second letter refers to *solution method*. The codes used in the study are explained below.

II. The participant made a wrong detection by regarding the mistake as correct, and explained the answer in detail.

CC. The participant found the mistake in students' answers and made a correct detection. The participant also explained this mistake successfully as well as stated the correct answer.

IB. The participant made a wrong detection by regarding the mistake as correct and did not make further explanations.

CI. The participant found the mistake in students' answers and made a correct detection; however, failed to provide a correct solution.

Validity and Reliability

The content validity of the data collection tool was ensured using a table of identifications developed by the researchers. The questions were chosen from the concepts about which the students had the most misconceptions, and different concepts were included using the table of identifications. First of all, questions were prepared separately from each other by two researchers using this table. Afterwards, these questions were discussed by the researchers and the five most appropriate questions that provided errors and misconceptions were selected. In order to ensure the content validity of these questions, the questions were shown to two different experts and the questions were corrected in line with their suggestions (Creswell, 2013; Houser, 2015; Streubert & Carpenter, 2011). In order to ensure reliability, two

Table 1. Categories and Codes Used in Data Analysis.

	Incorrect detection	Correct detection
Correct solution	IC	CC
Incorrect solution	II	CI
No answer	IN	CN

expert researchers independently coded the data at different times and locations. Inter-coder reliability performed at different times were calculated using the following formula developed by Miles and Huberman (1994): Reliability = number of agreements/number of agreements + disagreement × 100. After coding, disagreements were resolved by discussion and consensus. As a result, the reliability coefficient was calculated as .90.

Findings

In the study, five questions and student answers to these questions were given to the participants as a data collection tool. The participants were expected to identify the mistakes, present the ways they deal with the mistake, and provide solution suggestions. The answers given by the participants were coded in accordance with the categories. The answers given for each question were presented in tables with frequency and percentage. In addition, a direct quotation was presented for each question in order to support findings. Below are the questions presented to the participants, along with the student responses, the correct solution to each question was explained immediately after the student's answer. The content analysis was performed on the basis of these correct solutions. In this sense, the answers given by the participants were categorized and explained in accordance with Table 1.

1. "***There are three fair two-sided checkers painted with one side blue and the other red. When three checkers are tossed up at the same time, what is the probability that all three of them will be of the same color on the top?***"

Student response to the question above:

"***When three fair two-sided checkers are tossed up, at least two results will be the same. There is a 50%-50% chance that the third one will be blue or red. Accordingly, the probability of all three checkers being the same color is 1/2.***"

The mistake in the student's answer above is that while determining the sample space of the checkers tossed up at the same time, the sample space of two checkers was evaluated together and the other checker was treated as "third". In this question, it needs to be known that the realization event of the checkers is independent of each other, and accordingly, the sample space of the discrete events should be calculated independently. Since the probability of each checker to be the same color on the top is ½, the probability of three independent checkers being the same color needs to be determined as ½ × ½ × ½ = s1/8. The participants' answers to this question and the analysis of these answers are presented in Table 2.

As shown in Table 2, some of the participants (9.5%) stated that the student's answer was correct, the majority (78.5%) provide a correct solution, and a few (12%) suggested another incorrect solution.

A participant's answer in the "Correct detection, Incorrect solution" (CI) code was as follows:

S23: "*The student's answer is wrong. Since a checker has two sides, at least two of the three tossed checkers will have the same color on the tops. For example, let us suppose that the first tossed checker will be red and the second will be blue. In this case, whether the third checker will be blue or red, at least two of them will be of the same color.*"

S23 made a correct detection by stating that the student's answer was wrong. However, S23 failed to calculate the sample space of independent events successfully, and the probability of the first checker was treated in a fixed manner while the probabilities of the other checkers were treated as dependent.

Only 4 participants (9.5%) made an incorrect detection. It was found that they usually consider the student's answer as correct or provided another incorrect solution. A participant's answer in the "Incorrect detection, Incorrect solution" (II) code was as follows:

S12: "*Since the checker has two different sides, when the checker is tossed up, either red or blue color will appear on the top, and the probability of this is equal to 1/2. The possibilities as a result of tossing three checkers at the same time are as follows: RRR, RRB, RBB, BBB. Therefore, as the student said, at least 2 checkers must be in the same color. As it is a certain event, the probability that at least two checkers will be in the same color is one hundred percent. Thus, the last tossed checker will*

Table 2. Frequency and Percentages of Question 1.

Category	Code	Participation	*f*	%
Incorrect detection	II	S12, S16, S22, S40	4	9.5
Correct detection	CI	S2, S3, S7, S23, S41	5	12
	CC	S1, S4, S5, S6, S8, S9, S10, S11, S13, S14, S15, S17, S18, S19, S20, S21, S24, S25, S26, S27, S28, S29, S30, S31, S32, S33, S34, S35, S36, S37, S38, S39, S42	33	78.5
Total			42	100

determine the value of the probability. When two checkers will be red, the probability that the third checker is red will be 1/2, as the student answered successfully."

The participant made an incorrect detection by stating that the student's answer was correct. In addition, by explaining the student's answer in detail, S12 provided an incorrect solution. The error here was to evaluate the sample space of two of the three checker together and to consider the third checker independently from this group.

The examination of Table 2 indicated that certain pre-service teachers made incorrect detections and provided erroneous solutions. This observation can be attributed to their inadequate conceptual understanding of dependent and independent sample spaces.

2. ***When you roll 2 fair dice, what is the probability of the sum of the numbers on the top of the dice to be 8 or 10?***

Student response to this question was as follows:

"When two fair dice are rolled, the probability of rolling 8 or 10 is equal."

The main reason for the error in the above solution is the over-generalization that the numbers on the top of the dice have equal probability. However, an ordinary six-sided dice has the numbers from 1 to 6 placed on the faces, and the probability that the sum of the numbers on the top will be 8 or 10 is different from each other. The sample space for the probability of rolling two fair dice is 6^2, which equals to 36. There are 5 possibilities for the sum of the numbers on top of the dice to be 8: (6,2), (5,3), (4,4), (3,5), and (2,6). Consequently, the probability of having an 8 on the top must be 5/36. In addition, there are 3 possibilities for the sum of the numbers on top of the dice to be 10: (6,4), (5,5), and (4,6). Hence, the probability of having a 10 on the top must be 3/36. The participants' answers to this question and the analysis of the answers are presented in Table 3.

Table 3 showed that almost all of the participants (92.8%) detected the mistake in the student's answer and provided the correct solution.

In addition, 7.2% of the participants stated that the student's answer was wrong; however, they failed to provide a correct solution. A participant's answer in the "Correct detection, Incorrect solution" (CI;S8) code was as follows:

S8: "*The student has a misconception about the equiprobability bias. When two dice are rolled, the probability of having 8 as the sum is 1/6, and the probability of having 10 is 1/9. The two situations cannot be equal. The student may have thought the possibility of getting the numbers on the dice.*"

The participant made a correct detection by stating that the student's answer was wrong, failing to provide a correct solution. Here, similar to the student, the participant over-generalized by considering the probability of a single-digit number to be just like the other numbers on the dice surface. He/she believed that the probability of obtaining a two-digit number could be represented by one less than the total possible outcomes. Therefore, they argued that the probability of getting 8 and 10 could not be equal.

Table 3 showed that a significant portion of the participants provided correct detections and arrived at correct solutions. This success can be attributed to the fact that many probability questions are often based on dice, and individuals tend to encounter dice-related scenarios more frequently in daily life. On the other hand, those who provided incorrect solutions may have over-generalized the rules they were familiar with or failed to sufficiently consider the specific question at hand. This highlights the importance of educational materials in the learning process. Specifically, when it comes to probability problems that are commonly encountered in real-life situations, the use of appropriate materials can greatly support conceptual understanding.

3. "***There are 4 red, 3 blue, 2 yellow identical balls in a bag. How many different orders of the balls randomly selected from the bag without looking are possible?***"

Student response to this question was as follows:

"***Since there are 9 balls, there will be 9! orders.***"

The main mistake in the student's answer above is that the student disregarded that the balls are identical.

Table 3. Frequency and Percentages of Question 2.

Category	Code	Participation	*f*	%
	CI	S8, S16, S18	3	7.2
Correct detection	CC	S1, S2, S3, S4, S5, S6, S7, S9, S10, S11, S12, S13, S14, S15, S17, S19, S20, S21, S22, S23, S24, S25, S26, S27, S28, S29, S30, S31, S32, S33, S34, S35, S36, S37, S38, S39, S40, S41, S42	39	92.8
Total			42	100

The student regarded all the balls as different and treated them as if there was a total of 9 balls. However, it is stated in the question that the balls in the bag are identical in their own color categories. therefore, the balls are required to be ordered on the basis of the permutations with the repetition rule. In this sense, the answer is 9! / (4!.3!.2!). The participants' answers to this question and the analysis of the answers are presented in Table 4.

As seen in Table 4, 28.5% of the participants considered the student's answer as correct and provided an in-depth explanation for it. The answers of S42 in the "Incorrect detection, Incorrect solution" (II) code was as follows:

S42: "*The student's answer is correct. Since there is no grouping, separation, removal, or putting back in the question, the order of the 9 balls will be 9!.*"

S42 participants made an incorrect detection by considering the student's answer as correct. In addition, they provided an incorrect solution by confirming the student's answer. The participants ignored that the balls were identical, or did not take the fact balls were identical into account and provided a simple order.

In the third question, most of the participants (71.5%) stated that the student's answer was wrong and made a correct detection. It was found that 11.9% of these participants answered the question with another mistake (CI), 4.8% did not provide a solution (CN), and 54.8% provided a correct solution (CC). The participants' answers in the Correct detection category (CI; S34) was as follows:

S34: "*The student answered incorrectly as he or she did not treat the balls among themselves. If all the balls are drawn at the same time, we find how many different orders the balls have as follows*:

R R R R = 4! B B B = 3! Y Y = 2!

We get 3! from 3 different color categories. In this case, the answer will be 4!.3!.2!.3! = 24.6.2.6 = 1728."

S34 made a correct detection by stating that the student's answer was incorrect. However, he or she ignored that the balls are identical and provided an order by treating each ball in its own color category. Therefore, the participant provided an incorrect solution.

Table 4 indicated that the participants made more errors compared to the previous questions. This can be attributed to an overgeneralization of the simple sorting operation used in permutations. In the simple sorting process, the ordering is determined by the factorial of the number of objects, as each object occupies a unique position in the sorting order. However, when dealing with identical objects, their interchangeability reduces the change in position meaningless. This distinction needs to be perceived by individuals. Pre-service teachers who fail to grasp this difference may generalize the procedural knowledge they possess for a wide range of permutation questions. Once again, this underscores the significance of conceptual learning. In this regard, it is important to incorporate diverse objects in the teaching process of concepts to facilitate understanding.

4. ***"There are two identical wheels divided into two equal sections. The left section of the wheel is labeled as A and the right section as B. What is the probability of getting section A in both wheels through simultaneously spinning the arrows in the wheels?"***

Student response to this question was as follows:

"It is 50%. In other words, each wheel is divided into two equal sections, and the arrows have the same characteristics. The probability of getting section A on both wheels is ½."

The reason for the error in the student's answer above is due to the fact that the student regarded the wheels as independent events. While the probability of getting section A on both wheels is ½, the probability of getting the same section at the same time is ½ × ½ = ¼. The analysis of the participant's answers is presented in Table 5.

Table 5 showed that 7.2% of the participants considered the student's answer as correct and were included in the incorrect detection category. A sample excerpt in this category is given below.

S15: *"The student's answer is correct. Since the wheels are spinning in the same direction, we get section A in both wheels at the same time. In this case, there are two possibilities, either A or B, and in this case the probability of getting section A will be ½."*

Table 4. Frequency and Percentages of Question 3.

Category	Code	Participation	*f*	%
Incorrect detection	II	S9, S11, S12, S16, S19, S22, S23, S27, S28, S32, S36, S42	12	28.5
Correct detection	CI	S5, S7, S33, S34, S41	5	11.9
	CC	S1, S2, S3, S4, S6, S8, S10, S13, S14, S15, S17, S18, S20, S21, S24, S25, S29, S30, S35, S37, S38, S39, S40	23	54.8
	CN	S26, S31	2	4.8
Total			42	100

Considering that the student's answer is correct, S15 explained the solution. However, the participant provided an incorrect solution as s/he independently evaluated the probability of getting the same sections in the wheels at the same time.

On the other hand, most of the pre-service teachers (92.8%) stated that the student's answer was wrong and made a correct detection. It was revealed that 2.4% of these participants provided an incorrect solution, 2.4% did not provide a solution, and 88% provided a correct solution. The participants' answers in the Correct detection and, incorrect solution category (CI; S4) was as follows:

S4: "*The student's answer is wrong. The question asks the probability of getting an A on both wheels after spinning at the same time. When spun simultaneously in the same direction, the following possibilities will occur*:

- *We will get A in one and B in the other.*
- *We will get A in both.*
- *We will get B in both.*

Considering the probabilities, the probability that we will get A in both is 1 in 3. When one wheel is spun, the probability of getting an A is 50%. This is the answer given by the student. Therefore, the student's answer, that is ½, is wrong."

As seen above, S4 detected that the answer was wrong but provided an incorrect solution. Here, although the participants regarded the events as dependent, he or she failed to determine the sample space successfully.

Table 5 showed that the majority of the participants were able to make correct detections. This can be attributed to the fact that the question style is similar to scenarios encountered frequently in daily life problems. These types of questions, often found in games of chance or certain mobile games, contribute to the understanding that the events are dependent events. However, it should be noted that the participants who provided incorrect solutions may have mistakenly confused events that should occur simultaneously with independent events or may have over-generalized their procedural knowledge.

5. ***"There are three cards in a bag. both sides of one card are red (RR), both sides of the other is yellow (YY), and one side of the third card is red and yellow on the other (RY). Given that a randomly selected card from the bag is yellow on one side, what is the probability that the other side is yellow as well?"***

Student response to this question was as follows:

"It is ½ since if one side of the selected card is yellow, it cannot be RR. Thus, YY and RY remain. In this case, the probability of both sides being YY is 50%."

The student's answer above is wrong since he or she failed to determine the sample space successfully. The fact that one side of the card selected from the bag is yellow necessitates solving this question using conditional

Table 5. Frequency and Percentages of Question 4.

Category	Code	Participation	*f*	%
Incorrect detection	II	S15, S21, S28	3	7.2
Correct detection	CI	S4	1	2.4
	CC	S1, S2, S3, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S16, S17, S18, S19, S20, S22, S23, S24, S25, S26, S27, S29, S30, S31, S33, S34, S35, S36, S37, S38, S39, S40, S41, S42	37	88
	CN	S32	1	2.4
Total			42	100

Table 6. Frequency and Percentages of Question 5.

Category	Code	Participation	*f*	%
Incorrect detection	II	S1, S2, S3, S4, S6, S10, S11, S12, S15, S16, S18, S19, S21, S22, S24, S25, S26, S30, S31, S32, S33, S35, S38, S39, S40, S42	26	61.9
	IN	S8, S9	2	4.8
Correct detection	CI	S5, S7, S17, S23, S28, S29, S34, S36, S37, S41	10	23.7
	CC	S13, S14, S20, S27	4	9.6
Total			42	100

probability. In this case, the selected card is (RY) or (YY). However, since the sample space in the question will be (Y1, Y2), (Y2, Y1), and (RY), the answer should be 2/3. The analysis of the participant's answers is presented in Table 6.

As shown in Table 6, more than half of the participants (66.7%) considered the student's answer as correct and thus made an incorrect detection. In this regard, S1's answer is given below. In addition, 4.8% of the participants did not offer any further explanation and thus did not provide any solution. The participants' answers in the incorrect detection and incorrect solution category (II; S1) was as follows:

S1: "*Since one side of the selected card is yellow, the 2^nd^ or 3^rd^ card must be selected. If the 1^st^ card is selected, the probability of getting a yellow will be zero. This will be an exact event. In order for the other side to be yellow, the 2^nd^ Card must be selected. In this case, the specific result will be achieved.*

$$\textit{Result} = \frac{\textit{Specific card}}{\text{All events}} = \frac{2.\ \ \textit{card}}{2.\ \ \textit{or}\ 3.\ \ \textit{card}}$$

$$\frac{\binom{1}{1}}{\binom{2}{1}} = \frac{yy}{yy+yr} = \frac{1}{2}$$

The student's answer is correct."

S1 approved the answer given by the student above. Similar to the student, the participant, who determined the sample space as YY, YR, and RR, thought that the sample space became YY and RY, knowing that one side of the card was yellow. In this case, he/she believed that only one of the two cards he/she had chosen will be yellow on the other side.

On the other hand, 33.3% of the participants expressed that the answer was wrong and made a correct detection. Of them, 23.7% provided an incorrect solution, whereas 9.6% provided a correct solution. The participants' answers in the correct detection and incorrect solution category (CI; S28) was as follows:

S28: "*The student's answer is incorrect. The reason is that the sample space is determined incorrectly as a result of considering that each side of the cards is independent of the cards. However, the specific situation stated in the question must be taken into account. The conditions given in the student's solution were ignored. The given conditions were ignored in the student's solution. In addition, one of the 3 cards given in the question, which should have been eliminated, was not eliminated and was not included in the calculation. The reason for this is that the sides of the cards are treated independently of the cards. Sample space is as follows: RR, YY, RY.*

If one side of the selected card is yellow, this applies to both cards. We calculate this as $\binom{3}{2}$*. 2. Card has 2 yellow sides; 3. Card has 1 yellow side* $\left(\binom{3}{2}\right)$*. Since there are 6 sides In the whole event it will be 6. In this case, the result will be as follows:* $= \frac{\binom{3}{2}}{6} = \frac{\textit{intended stated}}{\textit{All stated}} = \frac{1}{2}$"

The participant mentioned above correctly identified the error in the answer but made an overgeneralization by conflating the concepts of probability and sample space determination.

Conditional probability questions are less commonly encountered compared to other question types. Therefore, it is crucial to utilize teaching materials during the instructional process to effectively explain the distinction between conditional probability and other types of probability questions. In addition, a structured approach should be adopted to facilitate conceptual learning in this area.

Discussion and Conclusion

It was found in the present study that the participants had a tendency to generalize procedural knowledge acquired in other concepts. The topic of probability holds significant importance in dealing with real-life problems and enhancing academic achievement. Therefore, it is crucial to employ diverse materials that facilitate conceptual learning. In order for students to comprehend the distinctions between dependent and independent events, probability and conditional probability concepts, as well as permutation and repeated permutation, it is necessary for them to engage in experiential and practical activities. Similarly, Şafak (2016) identified student misconceptions about dependent-independent events in probability. Ruz et al. (2021) observed in their study that preservice teachers were less successful in probability compared to other mathematics topics, and only one in five were able to provide correct answers to probability questions. Similar results were observed in different studies which concluded that preservice teachers had difficulties in understanding basic concepts about Probability (Arıcan & Kuzu, 2020; Garfield & Ben-Zvi, 2008, 2020; Koparan, 2019).

In this study, preservice mathematics teachers were asked to detect student mistakes in probability questions. In this regard, the participants' perspectives on the solutions and their ability to detect the mistake were investigated. In the first question, 21.5% of the participants provided incorrect solution. The reason for this finding is due to the fact that they confused the occurrence of discrete events. Similarly, there are different studies in the literature such representing the sample space differently (Shaughnessy & Ciancetta, 2002), negative or positive sequential effect, which includes the low probability of the same event occurring in a row, misconceptions about simple and compound events (Kazak, 2014), the idea that an event independent of another event can change the probability and ignoring the sample size (Fischbein & Schnarch, 1997; Garfield & Ahlgren, 1988; Li & Pereira-Mendoza, 2002; Memnun, 2008). Sample space determination is considered as an important stage in Probability (Fırat et al., 2016; Jones et al., 1999; Memnun, 2008). The utilization of appropriate materials and instructional practices that foster conceptual learning is essential to prevent the formation of misconceptions.

In the second question, it was observed that 7.2% of the participants provided incorrect solutions. It can be argued that providing incorrect solutions occurs as a result of student experiences, lack of infrastructure, and supplementary materials in the classroom (Koparan & Kaleli Yılmaz, 2015). It can be stated that mathematical concepts linked to real-life problems contribute to individuals' enduring learning and knowledge construction. Additionally, within the scope of the research, it was observed that some teacher candidates had misconceptions regarding certain concepts. Addressing and rectifying these misconceptions hold significance for the education of future generations.

In the third question, it was observed that 40.4% of the participants provided incorrect solutions. The reason for the mistake in this question is that the balls were not considered as identical and accordingly that the ordering rule of distinct objects was applied. It can be argued that the mathematical language that is not used or adequately perceived by the preservice teachers may pave the way to mistakes. Amir and Williams (1999), Jones et al. (1997), Sharma (2012, 2015), and Fischbein et al. (1991) examined the effects of mathematical language and probabilistic thinking. In this sense, the present study underlines the importance of the effect of mathematical language, estimation skills, and content knowledge on probability. It is stated in the literature that the teacher's ability to use mathematical language is also important for learning Probability (Jones et al., 1999). In the third question, the commonly made mistake is disregarding the identical nature of the balls. It should be understood that balls of the same color will not create any change in the ordering. It is important to utilize instructional materials and provide experiences that highlight this concept.

The fourth question is a dependent probability question in which two wheels are rotated at the same time. A significant majority of the participants provided correct detections and correct solutions in this question type, which is commonly encountered in probability-related problems. It can be said that individuals' familiarity with this question type, either through daily life problems or frequently encountered game types, facilitates their ability to think correctly.

The fifth question was a conditional probability question. This question type is among those that are rarely experienced during the teaching and learning process. The analysis of this question revealed that determining the sample space of the conditional probability question is more difficult than the dependent or independent events (Demirci et al., 2017). The topic of probability, which is used in daily life problems such as selection, arrangement, and ranking, needs to be structured and experienced through materials. Furthermore, it is crucial to establish connections with real-life situations and encourage analysis in order to enhance understanding. Contrary to ordinary questions, conditional probability questions require an individual to use higher-order thinking skills such as reasoning and inference. Consistent with the present study, numerous studies have shown that reasoning on Probabilistic issues is difficult (Barnes, 1998; Batanero et al., 1996; Beckmann, 2002; Behr et al., 1983; Bezzina, 2004; Bulut, 1994, 2001; Carpenter et al., 1981, 1983; Dooren et al., 2003; Fischbein et al., 1991; Fischbein & Schnarch, 1997; Garfield & Ahlgren, 1988; Gürbüz, 2006b; Hansen et al., 1985; Kapadia, 1985; Lawrence, 1999; Memnun, 2008; Munisamy & Doraisamy, 1998; Pijls et al., 2007; Shaughnessy, 1992; Truran, 1985). In addition, Bakırcı (2014) revealed that primary school students were more successful in questions requiring procedural knowledge about Probability, had difficulties in explaining Probability concepts, and in questions that required open-ended problem solving and reasoning. In addition, Bayazıt (2013) found that students cannot use their critical and creative thinking skills properly and that they consider real-life problems completely as mathematical operations. The fact that open-ended questions require extra effort in the measurement and evaluation stage and are difficult to score decreases the frequency of their use. Compared to multiple-choice questions, open-ended questions provide a great advantage for teachers to measure different characteristics such as generalization, reasoning, association, communication, and problem-solving skills (Yan, 2005). Considering that the preservice teachers had a moderate level of problem solving skills and they did not develop appropriate problem solving strategies (Güner & Erbay, 2021), problem-solving activities should be given importance as in this study. It was reported that preservice teachers used visuals and diagrams in problem-solving activities (Özdemir et al., 2018). In this study, it was found that some students used figures and diagrams. In this way, students' problem-solving skills can be improved and their anxiety levels can be reduced (Karasel et al., 2010).

Since statistics and probability are abstract and difficult to understand (Gürbüz, 2008), student-centered teaching should be delivered. The choice of effective teaching methods such as problem-solving and the way of analyzing questions is quite important in terms of understanding issues in Probability. In this context, the knowledge and teaching skills of the teacher, the ability to use appropriate teaching methods (Koparan & Kaleli-Yılmaz, 2015), the use of repetitions, their attitudes and behaviors toward students, and the way they deal with the student answers will have a significant effect on learning. In addition, lack of appropriate teaching materials (Gürbüz, 2006a), teacher-centered classroom environment (Yücel & Özkan, 2011), the teacher's lack of knowledge and skills (Bulut, 2001) are among the important reasons for Probability to be perceived as difficult. Teacher competency requires both content knowledge and effective use of teaching methods and techniques (Ball, 1990; Shulman, 1986; Wilson et al., 1987). At this point, the teacher competence factor becomes significant. In fact, there are different studies in the literature which revealed the effect of teacher efficacy on student achievement (Cakan, 2004; Dursun & Dede, 2004; Evirgen & Yıldız-İki Kardes, 2019; Rosenholtz, 1985; Seferoğlu, 2001; Turanlı et al., 2008). Furthermore, many studies have supported that content knowledge is highly effective on students' success (Ball, 1988, 1990; Ball & McDiarmid, 1990; Davis & Simmt, 2006; Harbison & Hanushek, 1992; Hill et al., 2004, 2005; Mullens et al., 1996; Rowan et al., 1997; Rowland et al., 2005; Tchoshanov, 2011). There are also studies which indicated that teachers' positive attitudes and behaviors toward the topic and students influence student success (Aiken, 1970; Arun, 1998; Aşkar, 1986; Baykul, 1990; Ekizoğlu & Tezer, 2007; X. Ma & Kishor, 1997; Moralı & Saracaloğlu, 1995; Şener, 2001; Tağ, 2000).

Implications for Practice

The content knowledge and pedagogical content knowledge of the students at the faculty of education should be constantly monitored. Instead of pattern problems, problems that will develop students' mathematical reasoning skills should be included. In such problems, open-ended questions should be asked to make students think with different methods and to avoid rote learning. In addition, teaching the subject about real-life problems will positively affect the development of students' reasoning skills.

In teaching probability, conceptual knowledge should be given as much importance as procedural knowledge. It should be explained what a mathematical concept does not mean as well as what it means (Özmantar et al., 2008). Preservice teachers should be informed about the misconceptions about probability and the ways to eliminate them. In this context, they should be included in the teaching process by taking their ideas about their misconceptions and solution suggestions. Thus, preservice teachers' mathematical reasoning skills will be developed.

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All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards.

Informed Consent

Informed consent was obtained from all individual participants included in the study.

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The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Supplemental Material

Supplemental material for this article is available online.

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