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Isospin mixing and Fermi transitions for some deformed nuclei

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Abstract: We have tried to give a self-consistent explanation of isospin mixing and Fermi beta decays for $54 \leq Z \leq 68$ deformed nuclei. The effect of isospin impurity in nuclear ground states on Fermi beta transitions has been investigated considering the restoration of symmetry violations stemming from the mean field approximation. The isobar analogue excitations in neighbor odd-odd nuclei have been obtained by following the proton-neutron quasiparticle random phase approximation (pnQRPA) procedure. The dependence of isospin admixture probability on the deformation parameter has been determined for various isotopes of Z = 54–68 nuclei. Then the variations of isobar analogue state energies and the β decay log(ft) values with deformation parameter have been calculated for the same nuclei. Thus, the influences of the isospin symmetry violations on Fermi transitions can be clearly understood.

Key words: Isospin symmetry, pnQRPA

1. Introduction

It is well known that the isospin symmetry is largely preserved by strong interactions. A small symmetry violation at the hadronic level is due to the difference in the masses of the up and down quarks [1]. In atomic nuclei, the main source of isospin symmetry breaking is the electromagnetic interaction [2,3]. The isovector and isotensor parts of electromagnetic force are much weaker than strong interactions between nucleons, so the effects of isospin symmetry breaking can be considered in a perturbative way. Hence, the formalism related to isospin is a very powerful concept in nuclear structure and reactions [4,5].

The investigation of Fermi beta decay is very important in understanding the isospin impurity in nuclear ground states. The problem of isospin mixing in nuclear ground states is of great importance in the experimental determination of the vector coupling constant of nucleon β-decay as well as in the description of the isobar analogue state energies and isospin multiplets [6–9]. The isospin mixing in nuclear ground states was studied in various models. The two-liquid hydrodynamic model was used to estimate the energies of the collective isovector monopole excitation with an isospin of $T = T_0 + 1$ [10]. The admixture of this excited state to the ground state with an isospin of $T = T_0$, caused by the Coulomb potential, turned out to be small (0.1–0.3) for all stable nuclei with $A > 40$. Quantitative estimates performed by using the shell model [11–13] are approximately one order of magnitude larger than the estimate of Bohr and Mottelson. There are several factors that may cause such a difference in the mentioned estimates. First, the shell model calculations were performed in the particle-hole approximation and used a limited number of configurations. Second, the residual isovector interaction

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was either neglected or included in the Tamm–Dankoff approximation. Third, the residual interaction was not related to the shell model potential in a self-consistent way. The isospin mixing for the proton-rich nuclei between A = 80 and A = 100 was calculated [14–24]. The Hartree–Fock calculations for these nuclei predicted the isospin mixing in the order of 0.03–0.05 [17]. The isospin mixing increased by an amount of 0.15–0.20 within the random phase approximation [14].

As known, the symmetry properties of the total nucleus Hamiltonian are violated in the mean field approximation. These violations have remarkable effects on the single and double beta decay rates [25–32]. The observation of Fermi β^+ decay in neutron-excess nuclei can be attributed to the isospin symmetry violations stemming from the electromagnetic interactions. In other words, the total β^+ decay probability, which is directly related to the isospin impurity in the ground state, would be zero if the isospin symmetry was an exact symmetry of the total nucleus Hamiltonian. However, the symmetry violations in the mean field level of approximation lead to an increment in the total β^+ decay probability. Therefore, the restoration of these symmetry violations has importance in determining the beta decay properties. The calculation of Fermi transitions with and without Coulomb interaction may be an efficient way to understand the isospin symmetry breaking effects [25], but a more realistic approximation should be used to define the transition properties. The isospin symmetry violation in the mean field approximation should be eliminated in a self-consistent way. The investigation of these transitions within a self-consistent method that is used to restore the symmetry violations stemming from the mean field approximation ensures that we can understand the decay properties more clearly. The calculation of Fermi decays for the systems with different proton numbers helps us to understand isospin symmetry breaking effects on these decays. The calculation of these decays for different deformation parameters clearly exhibits the symmetry breaking effects due to the sensitivity of the isospin impurity in nuclear ground states to this parameter. The isospin violation in the mean field level for neutron-excess nuclei was eliminated within the nuclear density functional theory [27]. A significant dependence of the magnitude of isospin breaking on the parametrization of the nuclear interaction term was found by the Warsaw group. Hence, the nucleon-nucleon residual interaction potential should be included in such a way that the isospin symmetry violation in the mean field level is restored. Pyatov's restoration method is an efficient way for such a restoration [26,31–38]. This method was used to investigate the isospin mixing and isobar analogue states for spherical nuclei [26,37,38]. Here Pyatov's method is applied for the investigation of isospin impurities and Fermi beta decays of various deformed nuclei ($54 \leq Z \leq 68$). There have been no theoretical analyses or experimental data on the isospin mixing ratios and Fermi transitions for these nuclei. A self-consistent explanation of isospin mixing and Fermi decays for the nuclei is given in the present work. In this respect, the symmetry violations that originate from the mean field approximation have been eliminated using Pyatov's restoration method and the isobar analogue excitations in neighbor odd-odd nuclei have been described without using any adjustable parameter. The probabilities for isospin mixing in the ground state of parent nuclei have been determined for the nuclei under consideration. The variation of the isospin admixture probability with the deformation parameter has been calculated and then the dependence of isobar analogue state energies and β-decay log(ft) values on the same parameter has been studied. Thus, the influences of isospin impurity in the ground state of the parent nucleus on Fermi transitions can be understood. The details of restoration are given in the next section. Section 3 contains the calculated results for the isospin mixing and Fermi beta transitions for deformed Z = 54–68 nuclei.

2. Method

The Fermi β-decay operators are described as:

$$T^{+}=\sum_{i=1}^{A} t_{+}(i) \tag{1}$$

$$T^{-}=\left(T^{+}\right)^{\dagger}.$$

Let us define a new charge exchange operator as below:

$$F^{\rho}=\frac{1}{2}\left(T^{+}+\rho T^{-}\right) \ (\rho=\pm). \tag{2}$$

The single quasiparticle Hamiltonian is defined as follows:

$$H_{sqp}=\sum_{s\sigma} \varepsilon_{s}(\tau)\, \alpha_{s\sigma}^{\dagger}(\tau)\, \alpha_{s\sigma}(\tau) \ (\tau=n, p), \tag{3}$$

where $\varepsilon_{s}(\tau)$ is the single quasiparticle (sqp) energy, and $\alpha_{s\sigma}^{\dagger}(\tau)$ $(\alpha_{s\sigma}(\tau))$ is the quasiparticle creation (annihilation) operator. Let us note that the n and p abbreviations are used instead of s_n and s_p, respectively. The nuclear part of the total Hamiltonian must commute with the Fermi β-decay operator

$$\left[H-V_{C}, F^{\rho}\right]=0, \tag{4}$$

but this commutativity is broken in the mean field approximation as below:

$$\left[H_{sqp}-V_{C}, F^{\rho}\right] \neq 0, \tag{5}$$

where Vc is the Coulomb term of a deformed Woods–Saxon potential [39].

The isospin invariance of the nuclear part broken in the mean field level of the approximation can be restored by including an effective interaction potential defined as below:

$$h=\sum_{\rho} \frac{1}{4 \gamma_{\rho}}\left[H_{sqp}-V_{C}, F^{\rho}\right]^{\dagger} \bullet\left[H_{sqp}-V_{C}, F^{\rho}\right]. \tag{6}$$

The effective interaction constant is found from the following condition,

$$\left[H_{sqp}-V_{C}+h, F^{\rho}\right]=0, \tag{7}$$

and taken out to be a free parameter.

$$\gamma_{\rho}=\langle 0|\left[\left[H_{sqp}-V_{C}+h, F^{\rho}\right], F^{\rho}\right]|0\rangle. \tag{8}$$

The pnQRPA Hamiltonian for the investigation of the isobar analogue states is described as follows:

$$H=H_{sqp}+h. \tag{9}$$

The phonon creation operator for the isobar analogue states is described as:

$$|i\rangle=Q_{i}^{\dagger}|0\rangle=\sum_{np}\left[\psi_{np}^{i} C_{np}^{\dagger}+\varphi_{np}^{i} C_{np}\right]|0\rangle, \tag{10}$$

where $C_{np}^{\dagger}$ (C_{np}) is the quasiboson creation (annihilation) operator, and ψ_{np}^{i} and φ_{np}^{i} are the forward and backward amplitudes, respectively. The following equation is solved to obtain the energies and wave functions of the isobar analogue excitations:

$$\left[H_{sqp} + h, Q_i^{\dagger}\right] |0\rangle = \omega_i Q_i^{\dagger} |0\rangle , \tag{11}$$

where ω_i corresponds to the excitation energies for the isobaric analogue states.

The total β decay strength for Fermi transitions is given as follows:

$$S^{\pm} = \sum_i \left| M_{\beta^{\pm}}^{i} (0_{g.s}^{+} \to 0_i^{+}) \right|^2 , \tag{12}$$

$$M_{\beta^{\pm}}^{i} (0_{g.s}^{+} \to 0_i^{+}) = \langle 0 | \left[Q_i, T^{\pm} \right] | 0 \rangle .$$

These strengths must fulfill the sum rule in the following form:

$$S^{-} - S^{+} = N - Z. \tag{13}$$

The probability of isospin mixing in the ground state of the parent nucleus is calculated by using Eq. (14):

$$b^2 = \frac{1}{2(T_0 + 1)} \sum_i \left| M_{\beta^{+}}^{i} \right|^2 , \tag{14}$$

$$T_0 = \frac{N - Z}{2}.$$

3. Results and discussion

The isospin admixture probabilities in the ground states of deformed Z = 54–68 nuclei and Fermi β transitions are calculated by using Pyatov's restoration method within the framework of pnQRPA formalism. The Nilsson single particle energies and wave functions are calculated within the deformed Woods–Saxon potential [39,40]. All energy levels up to 8 MeV are considered for neutrons and protons. The proton and neutron pairing gaps are determined as $\mathbf{\Delta_p}=\mathbf{C_p}\big/\sqrt{\mathbf{A}}$ and $\mathbf{\Delta_n}=\mathbf{C_n}\big/\sqrt{\mathbf{A}}$ MeV, respectively [41]. The pairing strength parameters ($\mathbf{C_p}$ and $\mathbf{C_n}$) are chosen so that the experimental pairing gaps are reproduced [42]. The fixed values of pairing interaction and the corresponding β transition strengths are presented in the Table. The present deformations in the 2nd column are determined from the single particle energies as shown in [43].

The calculated probabilities for the isospin mixing in nuclear ground states are given in Figure 1. The upward and downward figures represent the results of the restoration method and a phenomenological formula [$b^2 = 3.5 \times 10^{-7} Z^2 A^{2/3} (T_0 + 1)^{-1}$] [41], respectively. The curves obtained in both methods almost exhibit the same behavior as Z and N, but the calculated probabilities within restoration are approximately three times larger than those obtained with the phenomenological formula. The difference between the present calculations and the phenomenological formula originates from the variation of β^{+} transition probability (see Eq. (14)) with proton and neutron number (see Figure 2). As expected, the β^{+} decay probability usually decreases with N and increases with Z. The fluctuations in curves can be attributed to the differences in deformation effects. In order to understand the deformation effects on the isospin impurities, the dependence of the isospin

Table. The calculated values of β^- and β^+ decay strengths.

Nuclei	β	C_n(MeV)	C_p(MeV)	S^-	S^+	S^- - S^+	N - Z
^{124}Xe	0.18	10.80	10.69	15.976	0.033	15.943	16
^{126}Xe	0.16	11.24	10.71	17.974	0.030	17.944	18
^{128}Xe	0.15	11.51	10.94	19.862	0.031	19.831	20
^{130}Xe	0.13	12.06	11.50	22.167	0.031	22.136	22
^{132}Xe	0.11	12.72	12.10	24.010	0.027	23.983	24
^{126}Ba	0.20	9.68	11.14	13.920	0.073	13.847	14
^{128}Ba	0.17	10.06	11.20	15.857	0.055	15.802	16
^{130}Ba	0.16	10.64	11.40	19.590	0.060	19.530	18
^{132}Ba	0.14	10.80	11.82	20.020	0.060	19.960	20
^{134}Ba	0.12	10.53	12.70	22.020	0.050	21.970	22
^{128}Ce	0.22	9.49	11.54	10.965	0.181	10.784	12
^{130}Ce	0.18	9.65	11.65	13.980	0.120	13.860	14
^{132}Ce	0.17	9.94	11.93	16.110	0.120	15.990	16
^{134}Ce	0.15	10.20	12.40	18.525	0.098	18.427	18
^{136}Ce	0.12	10.97	12.98	19.981	0.070	19.911	20
^{130}Nd	0.24	9.67	10.28	10.265	0.236	10.029	10
^{132}Nd	0.20	9.81	10.72	12.200	0.255	11.945	12
^{134}Nd	0.18	9.65	12.44	14.197	0.224	13.973	14
^{136}Nd	0.15	9.96	12.83	16.102	0.166	15.936	16
^{138}Nd	0.12	9.95	14.10	17.842	0.115	17.727	18
^{132}Sm	0.21	9.65	10.12	8.247	0.217	8.030	8
^{134}Sm	0.21	9.91	10.23	10.286	0.225	10.061	10
^{136}Sm	0.216	9.61	11.96	12.351	0.246	12.105	12
^{138}Sm	0.16	9.60	12.84	14.270	0.276	13.994	14
^{140}Sm	0.12	9.90	14.41	16.264	0.178	16.086	16
^{134}Gd	0.26	9.57	10.16	6.330	0.301	6.029	6
^{136}Gd	0.22	9.76	10.34	8.033	0.259	7.774	8
^{138}Gd	0.19	9.56	11.18	10.289	0.248	10.041	10
^{140}Gd	0.21	9.37	12.28	12.405	0.283	12.122	12
^{142}Gd	0.10	9.90	14.44	14.201	0.193	14.008	14
^{136}Dy	0.26	9.57	9.42	4.344	0.289	4.055	4
^{138}Dy	0.22	9.69	9.60	6.204	0.267	5.937	6
^{140}Dy	0.21	9.62	9.89	8.261	0.279	7.982	8
^{142}Dy	0.18	9.32	11.31	10.264	0.256	10.008	10
^{144}Dy	0.06	9.96	13.99	12.177	0.146	12.031	12
^{138}Er	0.25	9.64	8.14	2.369	0.270	2.099	2
^{140}Er	0.22	9.60	8.31	4.137	0.267	3.870	4
^{142}Er	0.20	9.65	8.94	6.140	0.278	5.862	6
^{144}Er	0.16	9.24	10.53	8.229	0.252	7.977	8
^{146}Er	0.16	9.97	12.63	10.292	0.242	10.050	10

admixture probability on the deformation parameter should be calculated for each isotope. Thus, the variation of the isospin mixing with the deformation parameter is presented in Figure 3. The isospin impurity in nuclear ground states originates from the variable part of Coulomb potential. In other words, the probability for the isospin mixing would be zero if Coulomb potential was constant in nuclear volume. In a deformed structure, the variable part of Coulomb potential is proportional to the quadratic part of the deformation parameter.

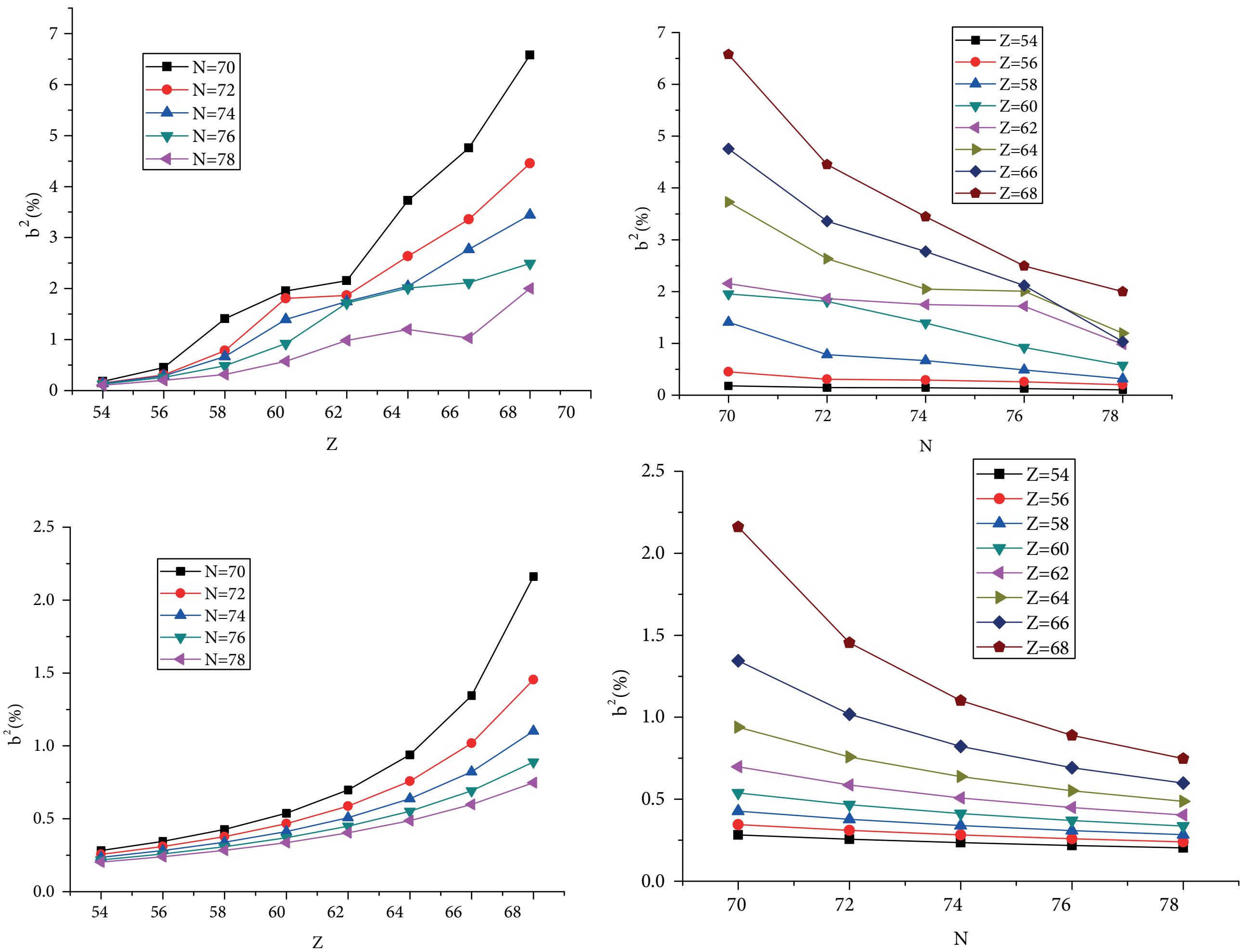

Figure 1. The variation of isospin admixture probability with the proton and neutron number.

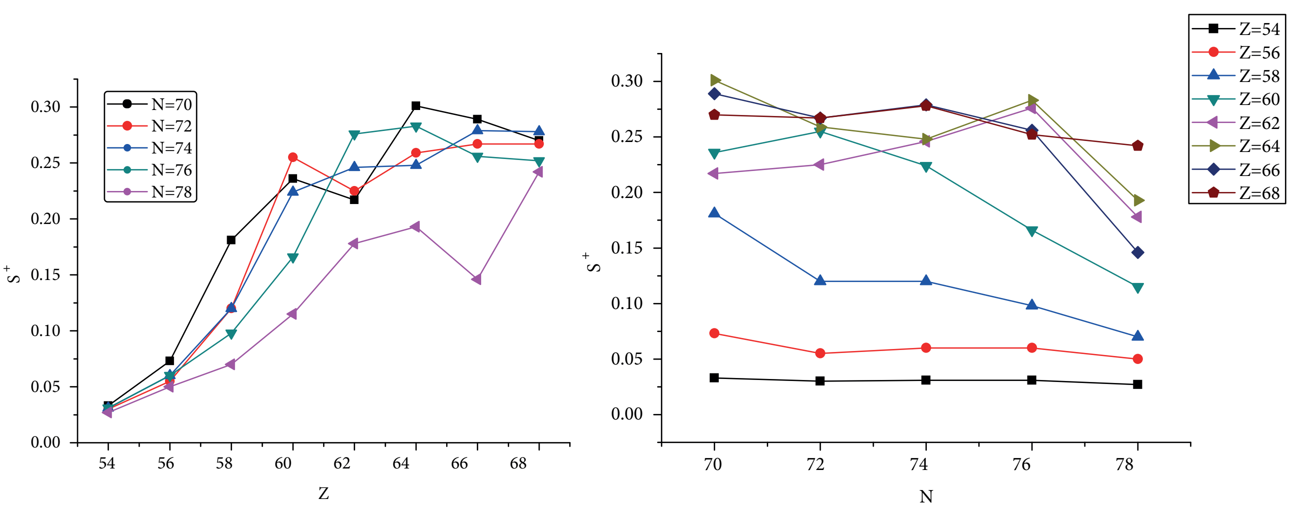

Figure 2. The variation of β^+ decay probability with the proton and neutron number.

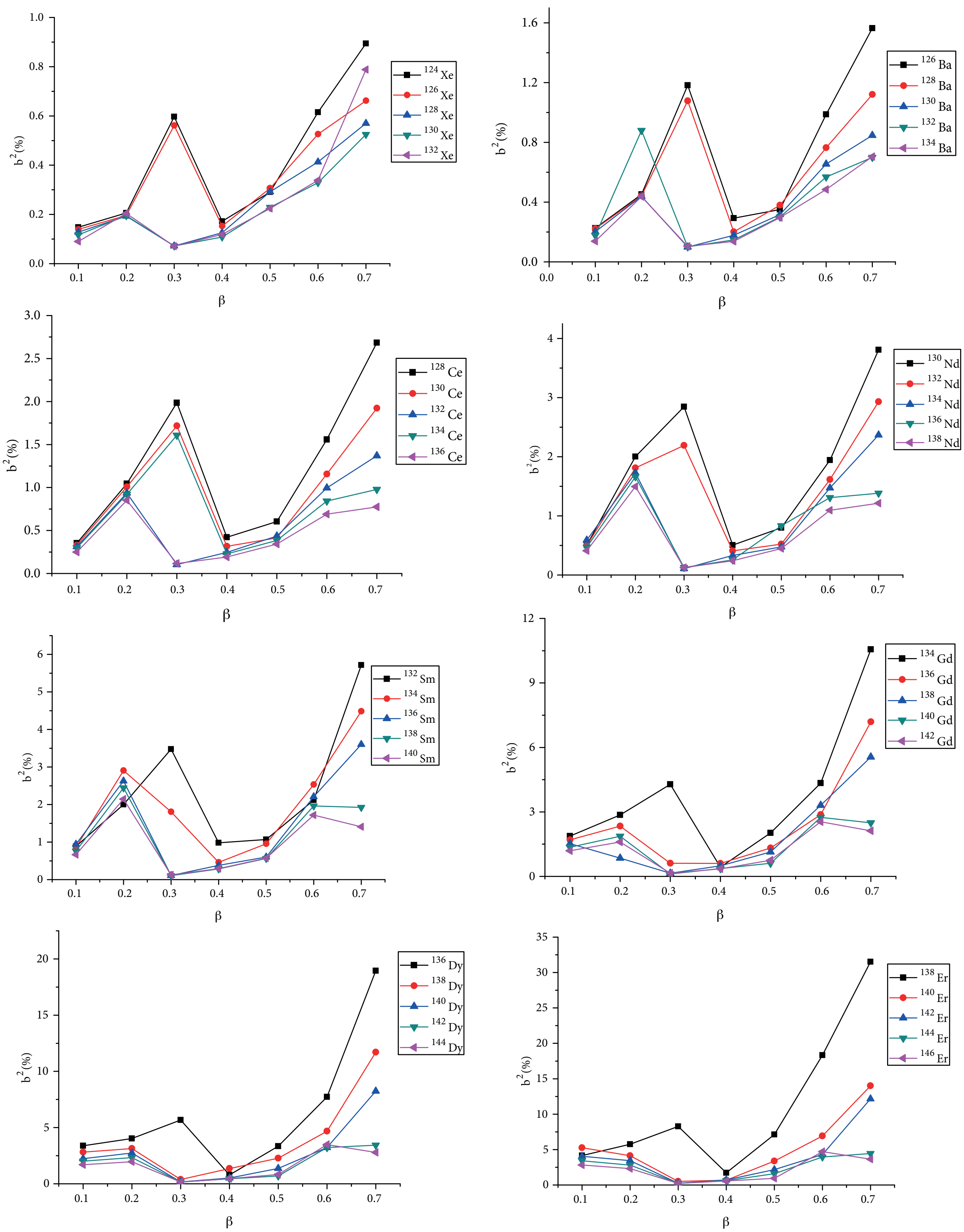

Figure 3. The dependence of isospin mixing on the deformation parameter.

Therefore, each curve in Figure 3 exhibits a quadratic character. The fluctuations in curves can be attributed to the variations in ground state configurations. Thus, it is well known that a small variation in the deformation parameter can lead to a sudden exchange of any two single-particle levels and the variation of ground state configurations. This case is also confirmed by the single-proton and neutron levels of the Nilsson potential [43]. The average energy values of the isobar analogue states in neighbor odd-odd nuclei and the β^- decay log(ft) values for the transitions between the initial ground state and the first isobar analogue state are presented in Figures 4–6. While the total Fermi decay probability shifts to a higher energy region for heavier isotones due to the repulsive effect of Coulomb interaction, the energy of the isobar analogue states shows no remarkable variation with the neutron number. The microscopic log(ft) values exhibit a more sensitive behavior to proton and neutron number due to the differences in the ground state configurations in comparison with the average energy values. In particular, the sensitivity of these configurations to the variations in deformation parameter can lead to a sudden variation of the β decay log(ft) values. Thus, the log(ft) values exhibit fluctuations with deformation parameter, as shown in Figure 7. However, the macroscopic energy values in Figure 5 show no important sensitivity to the variations of ground state configurations. It is seen from the figure that these energies usually exhibit a clear reduction with deformation parameter.

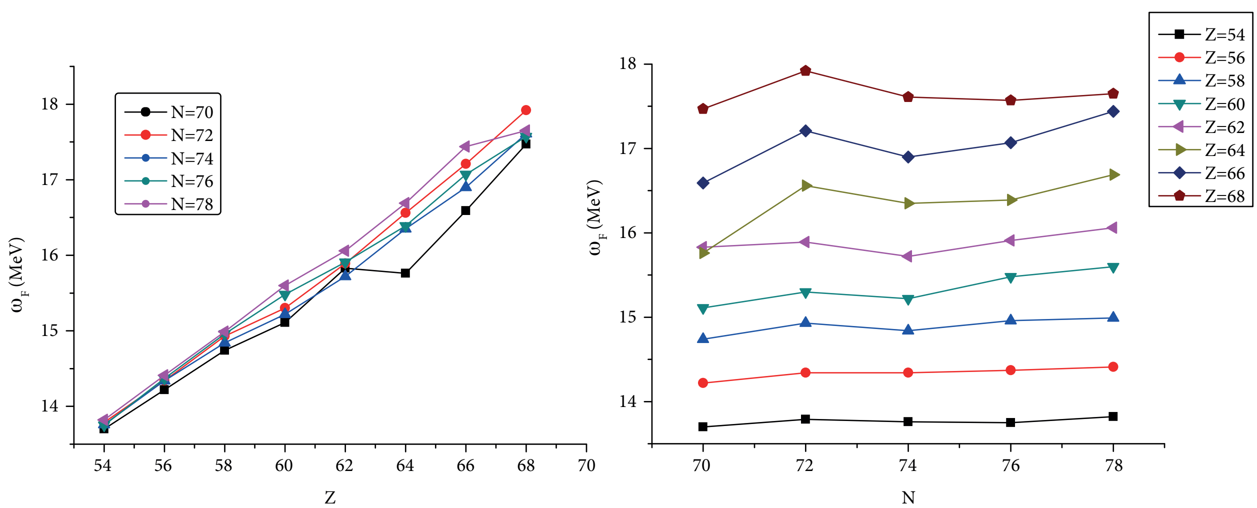

Figure 4. The variation of isobar analogue state energy with the proton and neutron number.

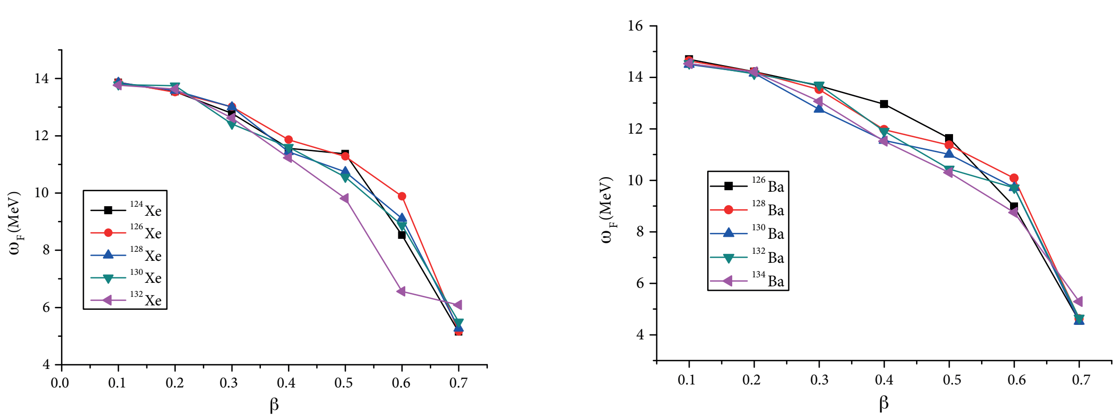

Figure 5. The dependence of isobar analogue state energy on the deformation parameter.

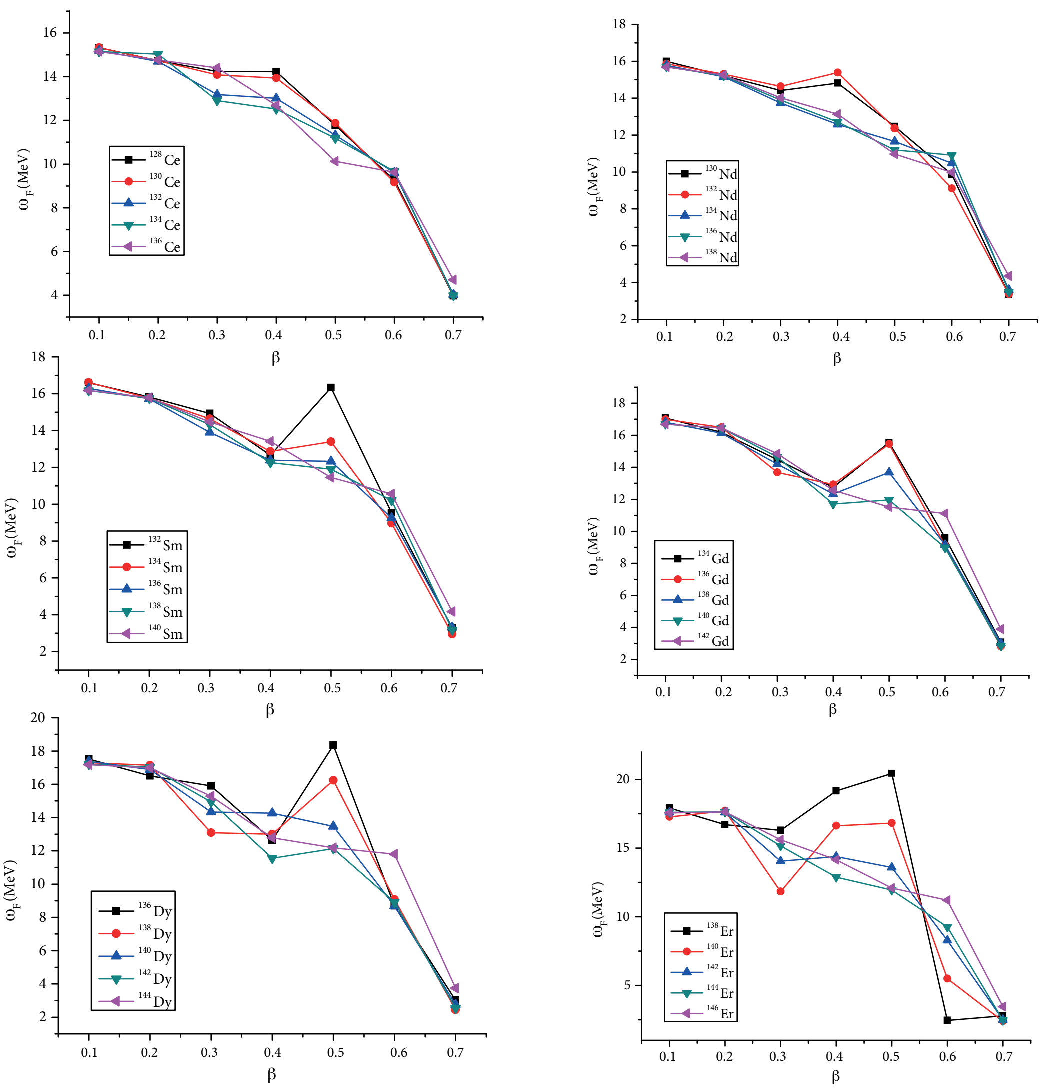

Figure 5. Continued.

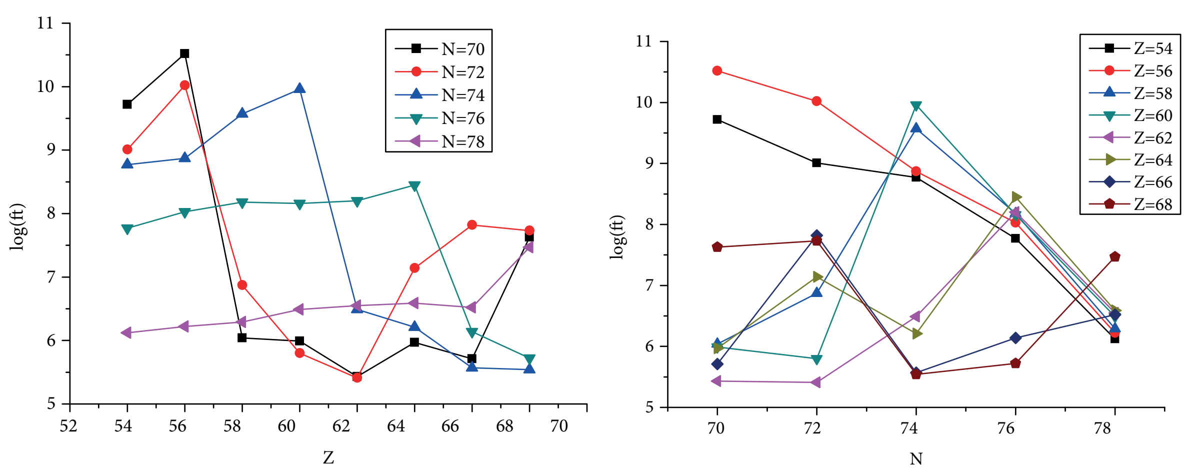

Figure 6. The variation of log(ft) value with the proton and neutron number.

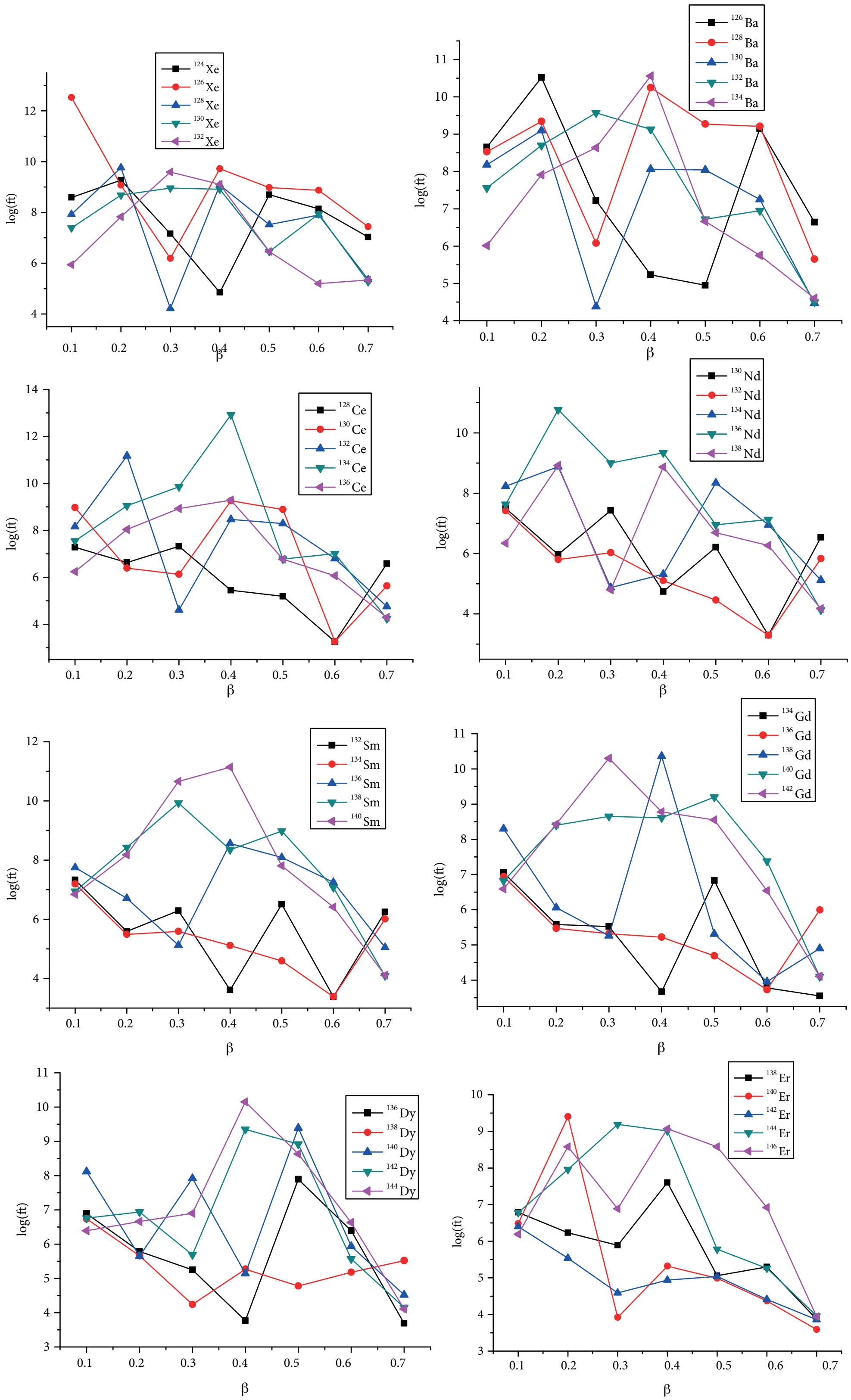

Figure 7. The dependence of log(ft) value on the deformation parameter.

4. Summary

The inclusion of nucleon-nucleon residual interactions leads to a remarkable quenching in β decay rates. This naturally affects the isospin mixing probability in nuclear ground states, which is directly proportional to the total β^+ decay probability. The magnitude of isospin symmetry violation shows an important dependence on the parametrization of nuclear interaction potential. Hence, the strength parameter of the effective interaction potential should be obtained from the isobaric invariance property of nuclear Hamiltonian (see Eq. (7)). Thus, it is possible to make a self-consistent explanation of isospin mixing ratios and isobar analogue states. The following conclusions can be drawn from the present calculations:

- Isospin mixing probabilities in the ground state of various deformed nuclei and Fermi β decays are calculated without using any adjustable parameter.
- There is not enough theoretical calculation and experimental data for the isospin impurity and β transitions of heavy deformed nuclei. The present calculations provide useful information about the isospin mixing and beta decay properties of $54 \leq Z \leq 68$ and $70 \leq N \leq 78$ nuclei.

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